Rewriting Flash Memories
Using Modern Coding Techniques

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Flash Memory in the Data Center

• An alternative to Hard Disk Drive (HDD)

• Flash advantages:
  – Fast access
  – Low power

• HDD advantages:
  – High capacity
Flash Memory Structure
Flash Cell

- Easy to add electrons
- Hard to remove: block erasure
- Erasures reduce reliability and speed
How to Increase Capacity?

• Use 8 levels instead of 4

• Problem: reliability and speed too low

• Proposed “solution”: reduce block erasures
Rewriting

0 → 1

• Write-Once Memory (WOM) Code
• Rivest, Shamir ‘82
  • for optical data storage

<table>
<thead>
<tr>
<th>Data to Store</th>
<th>1st-write code word</th>
<th>2nd-write code word</th>
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<tbody>
<tr>
<td>00</td>
<td>000</td>
<td>111</td>
</tr>
<tr>
<td>01</td>
<td>001</td>
<td>110</td>
</tr>
<tr>
<td>10</td>
<td>010</td>
<td>101</td>
</tr>
<tr>
<td>11</td>
<td>100</td>
<td>011</td>
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Cell values

Data

000
001
101
01
10
Literature

- **Memories with Defects**
  - Kuznetsov Tsybakov ’74, Belov Sashin ‘77, Gelfand Pinsker ’80

- **Optical Storage: WOM**

- **Flash Memories: WOM**
  - Jiang ’07, Yaakobi et al. ’12, Gabizon Shaltiel ’12, Jakovitz et al. ‘12, Burshtein Strugatski ’13 Shpilka ’13, Jiang et al. ’13

- WOM codes are still **not** used in flash products.
State Model

State vector = 10010

Word to write = 11110

• Model is adversarial or stochastic
Adversarial State Model

• Before the rewriting, the number of "written" cells is bounded.
• Rewriting always succeed.
• Most WOM codes are analyzed under this model.

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Stochastic State Model

• Will be used in this talk.

• The state of each cell is i.i.d (compressed).

• There is a chance that the rewriting will fail.
  – For example, if the state if 1111...1111

• The probability of rewriting failure should be small.
Capacity of WOM

• Capacity = tightest upper bound on the rate with arbitrarily low probability of rewriting failure.

• Setting:
  • Noiseless = what I read equal to what was written.
  • Binary alphabet
  • Two writes
  • \( s = \) state of a cell before rewriting
    • \( P(s=1) = \beta \)
  • Capacity = \( 1 - \beta \)
Outline

Two capacity-achieving WOM codes:

1. Message passing on sparse graphs
   - Practical algorithmic complexity
   - Easy circuit implementation

2. Entropy polarization
   - Easier to analyze
   - Modification of construction by Bushtein Strugatski ’13
   - Remove requirement for shared randomness
   - Add error correction
Sparse-Graph WOM Codes

• Joint work with W. Huang, Y. Li and J. Bruck at Caltech
• To be presented in ISIT 2015

State vector = 10010
Word to write = 11110
Construction

• Cosets of low-density-generator-matrix (LDGM) code
Construction

- **Cosets** of low-density-generator-matrix (LDGM) code

- Message-passing encoding (Martinian, Yedidia 2002 source coding)

  Encoder is given a **message** and a state = 101100

1. Find $z$ that represents the message. Say $z = 111010$

2. $z + s' = 0*01**$
Construction

- **Cosets** of low-density-generator-matrix (LDGM) code

- Message-passing encoding (Martinian, Yedidia 2002 source coding)

- **Encoder** is given a **message** and a state = 101100

1. Find \( z \) that represents the message. Say \( z = 111010 \)
2. \( z + s' = 0^*01^* \)
3. \( c = 000110 \)
4. \( c + z = 111100 \)

- \( c + z \) can be written over the state
When do we fail?

Theorem:
Failure = failure in iterative decoding of LDPC codes over BEC.

Implication:
1. Achieves the WOM capacity
2. Can use the same sparse graph of LDPC codes
Dealing with Failures

Property: rewriting failure only depends on the state
Does not depend on the message

Pre-checking
Capacity = 0.5
Reliability of Flash Memory

• What you read is not always what was written (errors).

• Raw Bit Error Rate (RBER) is typically above $10^{-5}$

• Applications require Uncorrectable Bit Error Rate (UBER) of $10^{-15}$
  – Achieved using BCH and LDPC codes
  – Typical blocklengths: 8k or 16k bits
Integration with Error Correction

Encoding:

- Code concatenation
- Reduces the rate of previous write

message + state
WOM code
writable vector
systematic ECC
redundancy bits
Integrated Code

• Find a good error correcting code (ECC) that contains the LDGM code.
• Use only cosets of the LDGM code that are contained in the ECC.
• Reminder: The LDGM code needs to be the dual of a good LDPC code.
• Need a good ECC that contains the dual of a good LDPC code.
Conjugate Code Pair

• Need a good ECC that contains the dual of a good LDPC code.
• Such pair of codes is also useful for protecting quantum bits.
• We found some known quantum LDPC codes that provide modest performance in our setting.
• Advantage: no need to reduce the rate of previous write.
Code Chaining

- Proposed in the context of broadcast coding
  - Hassani Urbanke ’13
Comparison of ECC approaches

• Setting:
  • Binary alphabet
  • 2 writes
  • BCH ECC
  • State probability = 0.5 (implies capacity = 0.5)
  • RBER = $1.3 \times 10^{-3}$
  • Rewriting failure rate $< 10^{-3}$
  • Blocklength $= \approx 8200$ bits

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<th>Code</th>
<th>UBER</th>
<th>Rate of 1st write</th>
<th>WOM Rate</th>
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<tr>
<td>Concatenated</td>
<td>$10^{-16}$</td>
<td>0.95</td>
<td>0.35</td>
</tr>
<tr>
<td>Conjugated</td>
<td>$10^{-5}$</td>
<td>1</td>
<td>0.21</td>
</tr>
<tr>
<td>Chained</td>
<td>$10^{-16}$</td>
<td>0.98</td>
<td>0.19</td>
</tr>
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Polar WOM Codes

• Joint work with Y. Li, J. Kliwer, M. Langberg, A. Jiang and J. Bruck
• Presented in ISIT 2014
Entropy Polarization

• Hadamard transform
  • Bijection

• Polarization Theorem (Arikan ‘09):
  1. $\tilde{x}$ is (almost) i.i.d. $\iff u_i | u_1 \ldots u_{i-1}$ is either
     • (Almost) deterministic, or
     • (Almost) uniform
  2. Fraction of uniform entries approaches the entropy of $x$

\[ G_n = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \otimes \log_2 n \]
\[ G_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \]

$$x_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} u_1$$
$$x_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} u_2$$
$$x_3 = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} u_3$$
$$x_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} u_4$$
Polar WOM Codes

- Polarization Theorem (Arikan ‘09):
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- WOM coding scheme: random variable $x | s$
  - $s =$ cell state before writing
  - $x =$ value to write
  - The random variable $x | s$ is distributed s.t. $P(x=0 | s=1) = 1$

$$
x_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$
WOM Coding Scheme

- WOM coding scheme: Consider $x \mid s$
  - $s =$ cell state before writing
  - $x =$ value to write
  - The random variable $x \mid s$ is distributed s.t. $P(x=0 \mid s=1) = 1$

- Iterative encoding, from 1 up to n: if $u_i \mid u_1 \ldots u_{i-1}, \bar{s}$ is deterministic
  - Assign $u_i$ accordingly
  - Else: assign $u_i$ with a message bit

- Decoding:
  1. Multiply the recorded vector by the Hadamard matrix to recover $\bar{u}$
  2. Recover the message from the “uniform” entries of $\bar{u}$

$$
\begin{align*}
x_1 & = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} u_1 \\
x_2 & = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} u_2 \\
x_3 & = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} u_3 \\
x_4 & = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} u_4 
\end{align*}
$$
Comparison

• Burshtein Strugatski ’13
  • First polar WOM codes
  • Based on a less mature polarization theorem
  • As a result, requires shared randomness

• Next step: integrate error correction
Generalized Setting

- Channel with encoder state information.
- Gelfand Pinsker ’80: Capacity
  \[
  C = \max_{p(v|s), x(v,s)} \left( I(V; Y) - I(V; S) \right)
  \]
- Theorem: \( V \rightarrow Y \rightarrow S \Rightarrow \) polar scheme achieves the channel capacity.
- Note: a similar result was established at the same time in the setting of broadcast channel
Noisy Write-Once Memory

- \( P(s=1) = \beta \)
- Capacity = \((1 - \beta)[1 - h(\alpha)]\)
  - Heegard '85
- Theorem: Model satisfies the Markov chain condition
Summary

1. Message passing on sparse graphs
   - Practical algorithmic complexity
   - Easy circuit implementation

2. Entropy polarization
   - Easier to analyze
   - Modification of construction by Bushtein Strugatski ’13
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