d-imbalance WOM codes for reduced inter-cell interference in Multi-level NVMs

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In a : Codes for Write Once Memories (WOM)

- In some storage devices, we can only write data to the device (and cannot erase)
- WOM codes enable us to perform multiple-writes
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- In some storage devices, we can only write data to the device (and cannot erase)
- WOM codes enable us to perform multiple-writes

First introduced by Rivest and Shamir (1982), aimed for

- Optical disks
- Punched card
- PROMS

IT researchers took over on 1985

Flash memories flood (2007-Now)
The “expensive” erase operation in flash

- Data is represented by the amount of electrical charge.
The “expensive” erase operation in flash

- Data is represented by the amount of electrical charge
- Cells can be written individually

'q-1' level

'0' level
The "expensive" erase operation in flash

- Data is represented by the amount of electrical charge
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\[ q \rightarrow q-1 \]

‘0’ level

‘q-1’ level
Data is represented by the amount of electrical charge

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The "expensive" erase operation in flash
The “expensive” erase operation in flash

- Data is represented by the amount of electrical charge
- Cells can be written individually

- ‘0’ level
- ‘q-1’ level
Data is represented by the amount of electrical charge.

Cells can be written individually.

However, can only be erased by erasing an entire block.
Write Once Memories codes enable to rewrite the block without erasing it

A $C(n,q,t,M)$ WOM code
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Number of memory cells
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A $C(n,q,t,M)$ WOM code

- Number of memory cells
- Number of memory levels
- Number of writes
- Write Once Memories codes enable to rewrite the block without erasing it

A $C(n,q,t,M)$ WOM code

- Number of memory cells
- Number of memory levels
- Number of writes
- Alphabet size of each write
WOM codes

- Write Once Memories codes enable to rewrite the block without erasing it

A $C(n,q,t,M)$ WOM code

- Number of memory cells
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- WOM codes have proven to theoretically reduce wear and increase the lifetime of the media
  - [Odeh&Cassuto, MSST ’14]
  - [Yadgar, Yaakobi, Schuster, FAST ’15]
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PRXY $q=8$

Standard FTL use
WOM codes

- Write Once Memories codes enable to rewrite the block without erasing it

A $C(n,q,t,M)$ WOM code

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WOM codes (in-place writes) increase inter-cell-interference
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**ICI - Example**

- **Breadth-first programming**

  Target values:

  5  2

  0  1  2  3  4  5  6

ICI - Example

- Breadth-first programming

Target values:

6
5
4
3
2
1
0

prog. step #1

Breadth-first programming

Target values: 5 2

ICI

Breadth-first programming

Target values: 5 2

prog. step #2

ICI - Example

- **Breadth-first programming**

  ![ICI Example Diagram]

  **Target values:**

  - 5
  - 2

Breadth-first programming

Target values: 5, 2

Breadth-first programming

Target values:

0 1 2 3 4 5 6

6
5
4
3
2
1
0

Breadth-first programming

Target values:

5

2

Reached its target level

Breadth-first programming

Target values: 5 2

Breadth-first programming

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Reached its target level

**Breadth-first programming**

Target values: 5 2

Reached its target level

Error!

Breadth-first programming

Minimizing level difference between adjacent cells can reduce ICI

Target values:

Error!

**Decoding Function: n=2**

\[ C_2 \]

\[
\begin{array}{ccc}
3 & 1 & 4 \\
2 & 3 & 6 & 0 & 2 \\
1 & 1 & 4 & 7 \\
0 & 0 & 2 & 5 \\
\end{array}
\]

\[ C_1 \]

\[ q=4 \]

\[ m \in \{0,1,\ldots,7\} \]

\[ M=8 \]

**C1:** Level of cell #1, **C2:** Level of cell #2

\[ \psi(c_1, c_2) = m \in \{0, \ldots, 7\} \]
We wish to write a sequence of three values: 3, 7, 1

Initial state
We wish to write a sequence of three values: 3, 7, 1
We wish to write a sequence of three values: 3,7,1

\[ |c_1 - c_2| = 2 \]

\[ |c_1 - c_2| = 3 \]
We wish to write a sequence of three values: 3, 7, 1
We wish to write a sequence of three values: 3,7,1

The difference between physical states is unconstrained

$$|c_1 - c_2| \leq d = q - 1$$
Research question:
Can we construct WOM codes that
1. bound the level differences, and
2. give many writes?

Related work: low level increases of individual cells [Qing Li, ‘11]
**Definition – d-imbalance WOM code:**
A d-imbalance WOM code \( C_d(n, q, t, M) \) guarantees that after each write the cell levels satisfy

\[
\max_{i,j} |c_i - c_j| \leq d
\]
The extreme case $d = \left\lceil \frac{n}{\sqrt{M}} \right \rceil - 1$: $n=2, M=8 \rightarrow d=2$
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$t = \left\lfloor \frac{q-1}{2} \right\rfloor$
writes
(optimal)
Maximally balanced
- Ex.: \(d = 2, \ t = \left\lceil \frac{q-1}{2} \right\rceil\)

Not balanced
- Ex.: \(d = q - 1, \ t = \left\lceil \frac{2(q-1)}{3} \right\rceil\)

Is there a middle ground?

\[
\frac{t}{q-1} = 0.5
\]

\[
\frac{t}{q-1} = 0.66\dot{\bar{6}}
\]
Main Result: Construction

Construction $- d_{\text{min}} + 1$-imbalance $n=2$ WOM code family:
For any $M = a^2 - 1$, $a$ integer, and any $q$, an explicit WOM code

$C_a(2, q, t, a^2 - 1)$ is constructed with $t = \left\lfloor \frac{3(q-1)}{3a-4} \right\rfloor$ writes.
Main Result: Construction

Construction – $d_{\text{min}}+1$-imbalance $n=2$ WOM code family:
For any $M = a^2 - 1$, $a$ integer, and any $q$, an explicit WOM code $C_a(2, q, t, a^2 - 1)$ is constructed with $t = \left\lfloor \frac{3(q-1)}{3a-4} \right\rfloor$ writes.

For $a = 3 \iff M = 8$:

\[
\frac{t}{q-1} = 0.5 \quad 0.6 \quad \frac{t}{q-1} = 0.66\dot{6}
\]
We wish to write a sequence of three values: 3, 7, 1
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It is guaranteed that through the entire write process $|c_1 - c_2| \leq d = 3$
Theorem: Upper bound \(d_{\text{min}} + 1\)-imbalance \(n=2\) WOM codes:

For \(M = 8\), the number of writes of any WOM code \(C_3(2, q, t, 8)\) is bounded by

\[
t \leq \left\lfloor \frac{3(q-1)}{5} \right\rfloor
\]
We start here
Sketch of proof

Total write sum = \(3\)

\[C_1\]

\[C_2\]
Sketch of proof

Total write sum = 3

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</table>

C₁

C₂
Sketch of proof

Total write sum = 3
Sketch of proof

Total write sum = 6
Sketch of proof

Total write sum = 6
Total write sum = 9

Sketch of proof

Total write sum = 3
Total write sum = 6
Total write sum = 9

\[ C_1 \]

\[ C_2 \]

Total write sum = 9
Sketch of proof

Total write sum = 9

\[ \sum_{i=1}^{n} C_i \]
Total write sum = 13

Must have write sum = 4
Sketch of proof

Total write sum = 13

- Last 3 writes needed write sum of 10
- \(2(q - 1) \geq 10t/3\)

Must have write sum 4

\[ C_1 \]

\[ C_2 \]
Construction is sharp!

For \( a = 3 \Leftrightarrow M = 8 \):

\[
\begin{align*}
\frac{t}{q-1} &= 0.5 \\
\frac{t}{q-1} &= 0.666
\end{align*}
\]

Optimal!
# Sample parameters, $M=8$

<table>
<thead>
<tr>
<th>$q$</th>
<th>$t$ - Upper bound d-unconstrained</th>
<th>$t$ - Construction 1 (d = 3)</th>
<th>$t$ - Construction (d = 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>4</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>16</td>
<td>9</td>
<td>9</td>
<td>7</td>
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<tr>
<td>20</td>
<td>12</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>32</td>
<td>20</td>
<td>18</td>
<td>15</td>
</tr>
</tbody>
</table>
Lattice-Based d-imbalance WOM codes
Definition – sum-rate:

\[ R_{\text{sum}} = \frac{\sum_{i=1}^{t} \log_2(M_i)}{n} \]

- **Main idea:**
  - Deriving optimal sum-rate solution in the continuous regime
  - Performing discretization process to obtain “nearly optimal” WOM code
Theorem [Bhatia et. al]:
The optimal continuous boundary between the writes of a 2-cell 2-write lattice-based WOM code is given by the following equation of an hyperbola:

\[ \beta(x) = q - 1 - \frac{\omega_2 (q - 1)^2}{q - 1 - x} \]

where

\[ \omega_2 = -\frac{1}{2} \left[ W_{-1} \left( \frac{-1}{2\sqrt{e}} \right) \right]^{-1} \]

A. Bhatia, and P.H. Siegel, “Multilevel 2-cell t-write Codes,” ITW 2012
Theorem [Bhatia et. al]:

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Real branch of the Lambert W function

A. Bhatia, and P.H. Siegel, “Multilevel 2-cell t-write Codes,” ITW 2012
A $C(n = 2, q = 8, t = 2, M = (24, 23))$ WOM code
Lattice Based WOM codes - Example

- A $C(n = 2, q = 8, t = 2, M = (24, 23))$ WOM code

Hyperbola
A \( C(n = 2, q = 8, t = 2, M = (24, 23)) \) WOM code
Log of areas $Z_1, Z_2$ are rates of write1, write2

Sketch of Proof: Optimization Problem

Maximize continuous sum-rate
Log of areas $Z_1, Z_2$ are rates of write1, write2

\[
\max_{\tilde{\beta}(x)} \left\{ \int_0^{x_{\text{sup}}} \tilde{\beta}(x) \, dx \cdot \min_{\forall x, \tilde{\beta}(x)} \left[ (q - 1 - x)(q - 1 - \tilde{\beta}(x)) \right] \right\}
\]

Maximize continuous sum-rate
Log of areas $Z_1, Z_2$ are rates of write1, write2

Sketch of Proof: Optimization Problem

Maximize continuous sum-rate
Theorem:

When $d \leq \frac{3}{7} (q - 1)$, the optimal boundary for a maximal sum-rate $d$-imbalance lattice based WOM code is given by $\beta_d(x)$ satisfying:

$$\frac{(q - 1 - x)(q - 1 - \beta_d(x)) - (q - 1 - d - x)^2}{2} \text{ or } \frac{(q - 1 - d - \beta_d(x))^2}{2} = d(q - 1) - \frac{5d^2}{6}$$
**Theorem:**

When \( d \leq \frac{3}{7} (q - 1) \), the optimal boundary for a maximal sum-rate \( d \)-imbalance lattice based WOM code is given by \( \beta_d(x) \) satisfying:

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This is actually a parabola…
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The optimal sum rate:

\[
R_{sum} = \log_2 \left[ d (q - 1) - \frac{5d^2}{6} \right]
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Theorem:

When \( d \leq \frac{3}{7}(q - 1) \), the optimal boundary for a maximal sum-rate \textbf{d-imbalance} lattice based WOM code is given by \( \beta_d(x) \) satisfying:

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This is actually a parabola…

The optimal sum rate:

\[
R_{\text{sum}} = \log_2 \left[ d(q - 1) - \frac{5d^2}{6} \right]
\]

This is a \textbf{fixed rate} code
The optimization problem becomes
The optimization problem becomes
The optimization problem becomes

Hyperbola of the unconstrained model
The optimization problem becomes

\[ y = x + d \]

\[ y = x - d \]

Parabola of the d-imbalance model
We first show that the boundary is given by:

\[
\frac{(q - 1 - x)(q - 1 - \beta_d(x)) - (q - 1 - d - x)^2}{2} - \frac{(q - 1 - d - \beta_d(x))^2}{2} = Z_2
\]

Then, we find the optimal parameters of the boundary function, by solving:

\[
\frac{Z_1}{2} = \int_{0}^{q - \frac{Z_2}{2d}} \beta_d(x) - x \, dx - \int_{0}^{q - \frac{Z_2}{2d} - \frac{5d}{4}} \beta_d(x) - (x + d) \, dx
\]

\[
= \int_{q - \frac{Z_2}{2d} - \frac{5d}{4}}^{q - \frac{Z_2}{2d} - \frac{d}{2}} \beta_d(x) - x \, dx + \int_{0}^{d} d \cdot dx.
\]
Continuous sum-rate comparison of 2-cell lattice-based WOM codes for q=8 and q=16:

<table>
<thead>
<tr>
<th>q</th>
<th>d</th>
<th>$R_{sum}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>-</td>
<td>3.97</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>3.75</td>
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<tr>
<td>8</td>
<td>2</td>
<td>3.42</td>
</tr>
<tr>
<td>16</td>
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<td>6.17</td>
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<tr>
<td>16</td>
<td>6</td>
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<tr>
<td>16</td>
<td>5</td>
<td>5.76</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
<td>5.54</td>
</tr>
<tr>
<td>16</td>
<td>3</td>
<td>5.29</td>
</tr>
</tbody>
</table>
Let us consider $C_{3\text{-imb}}(2, 8, 2, (Z_1, Z_2))$

The optimal continuous cardinalities are $(Z_1, Z_2) = (13.5, 13.5)$

However, the discrete cardinalities are $(M_1, M_2) = (18, 21)$
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| Sum-Rate of the unconstrained case | 4.55 |
| Sum-Rate of lattice based 3-imbalance | 4.28 |
| Sum-Rate of previous construction (fixed rate) | 3.91 |
ICI reduction – Test case
By minor adjustment to the update process, the d-imbalance property is kept through the entire wordline.
We analyze the worst case scenario ICI
We analyze the worst case scenario ICI vs. $q-1$ vs. $d$. 
We analyze the worst case scenario ICI.

By using the d-imbalance WOM codes, the worst case scenario BER can be reduced by

\[ \frac{BER_{d-imb}}{BER_{uncon.}} = \frac{Q \left( \frac{V_{ref} - \alpha \frac{d}{q-1} \Delta V}{\sigma} \right)}{Q \left( \frac{V_{ref} - \alpha \Delta V}{\sigma} \right)} \]

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Voltage shift of aggressor cell

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\]

- Capacitance coefficient
- Voltage shift of aggressor cell
- Voltage threshold for read
- STD of voltage distribution

ICI reduction – test case (cont.)

Programming alternate bitlines

Using d-imbalance WOM codes

ICI reduction – test case (cont.)

Many open questions

- Larger d
- n>2
- Other imbalance models
- Inter-page interference
Questions?