Sequence Reconstruction for Non-Identical Channels

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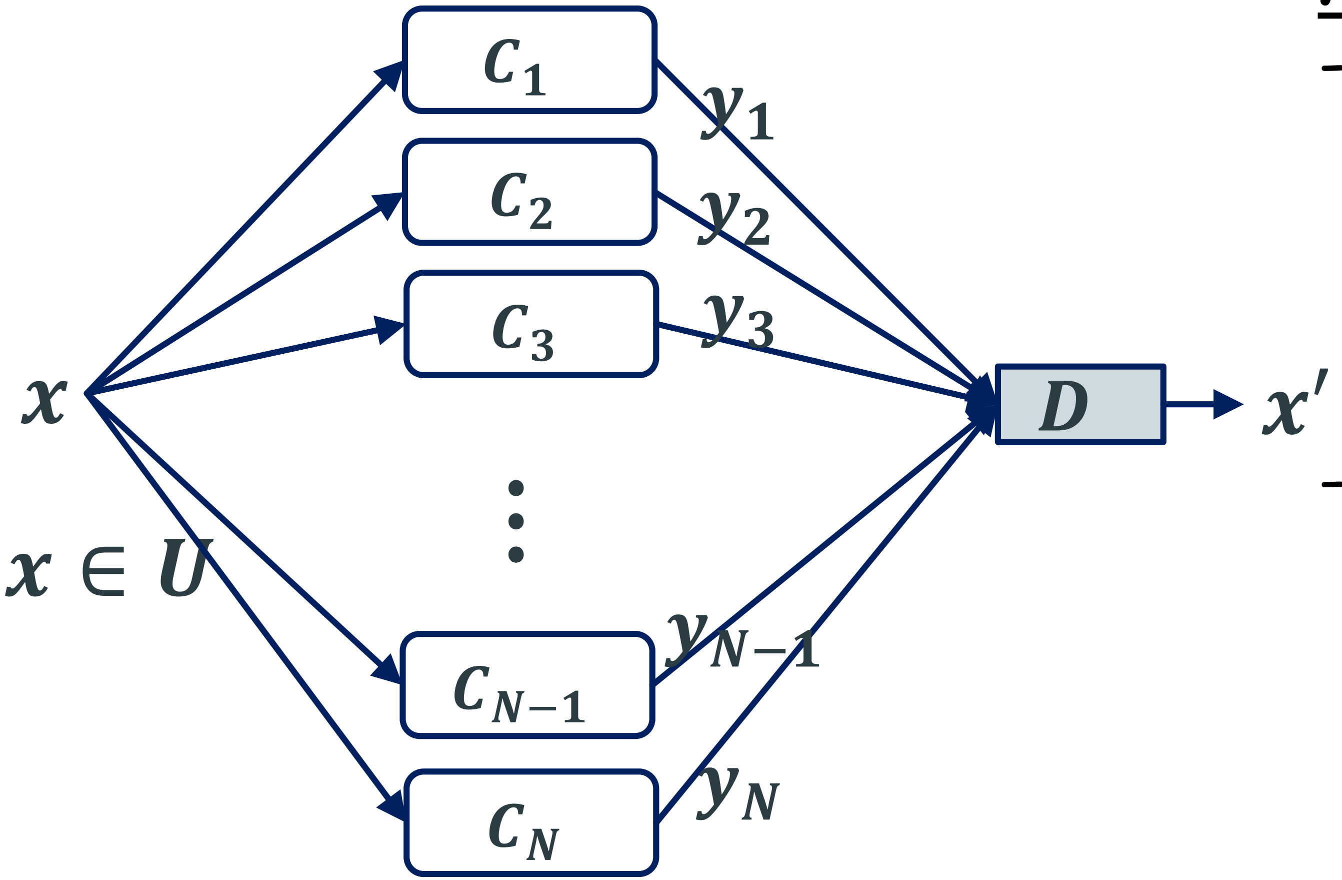
Sequence Reconstruction



Motivation:

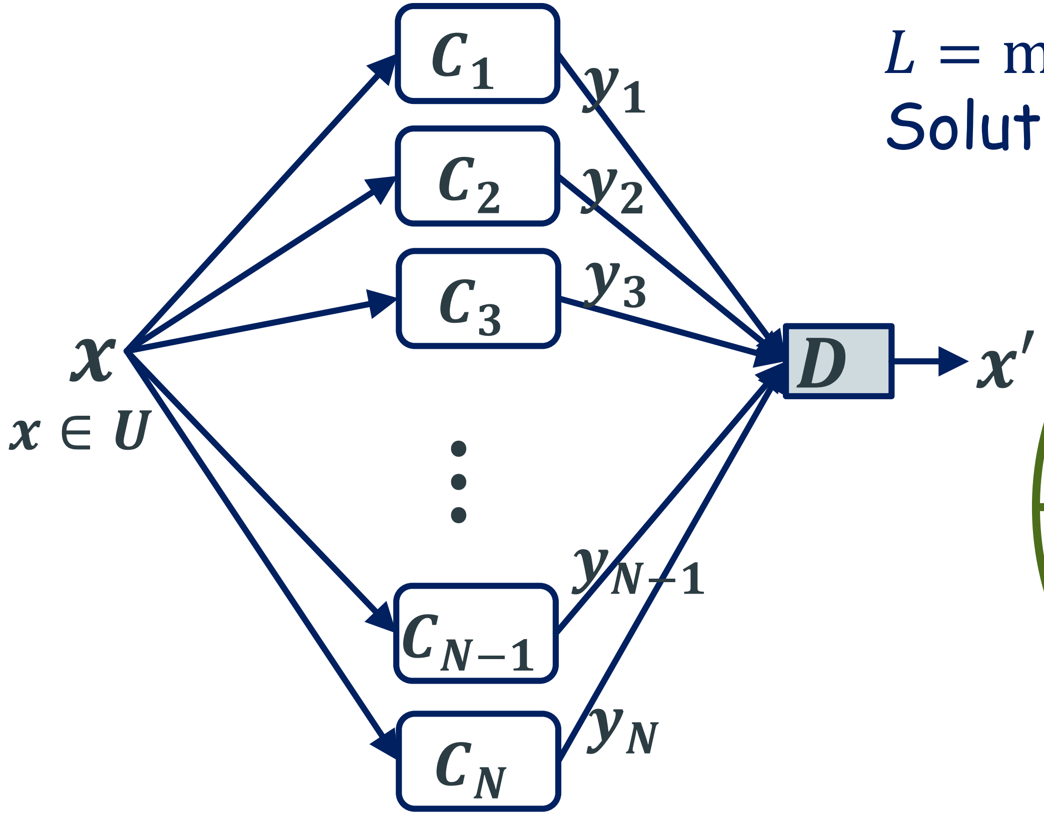
- Chemical and biological processes where the information is replicated and can be read from different noisy sources.
- Storage technologies, where the stored information has multiple copies or a single copy is read by several different read heads, e.g., DNA storage.

Levenshtein, "Efficient reconstruction of sequences," IEEE Trans. on Inform. Theory, 2001.

Sequence Reconstruction

A **channel system** of size N

A t-**error channel** –
a channel causes at most t erros.



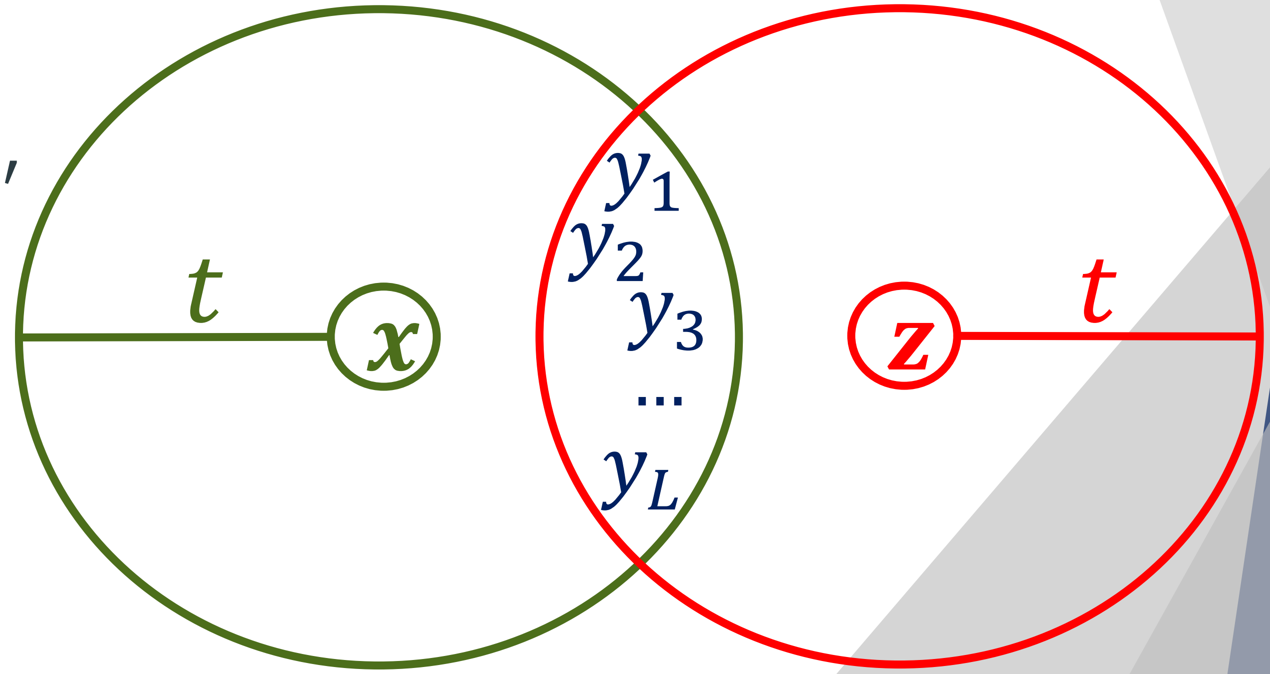
Goal: $\forall x \in U$: $x' = x$ - **Exact reconstruction.**

Given: U, the channels are t-error.

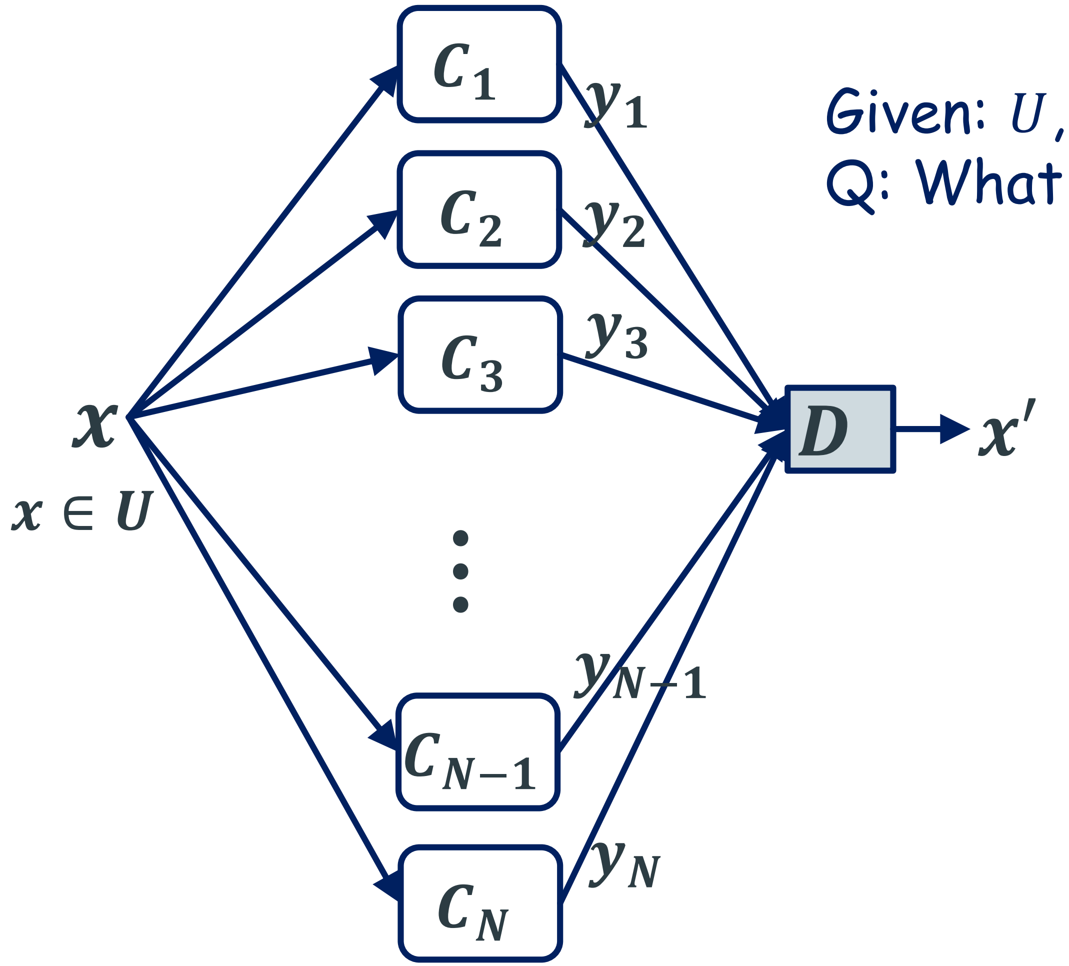
Q: What is the **minimal** N for exact reconstruction?

$L = \max\{|B_t(x) \cap B_t(z)| : x, z \in U\}$

Solution: $L + 1$ (Levenshtein'01)



Sequence Reconstruction



Goal: $\forall x \in U$: $x' = x$ - **Exact reconstruction**.

Given: U, data about the errors
Q: What is the **minimal** N for exact reconstruction?

Examples for data about the errors:

1. 1/2 of the channels are t_1-error, and 1/2 are t_2-error.
2. The first 1/2 of the channels are t_1-error, and the rest are t_2-error.
3. The average number of errors is given

Sequence Reconstruction References

- Problem presentation. substitution errors, Johnson graphs, deletions, insertions, and more general metric distances. Le'01.
- Some general error graphs

  LeKM'08, LeSi'09
- Permutations – K'07, K'08, KLeSi'07,
- Kendall's τ : YSwLaB'13
- Insertions – SGSD'15, GY'16
- Deletions –YG'16

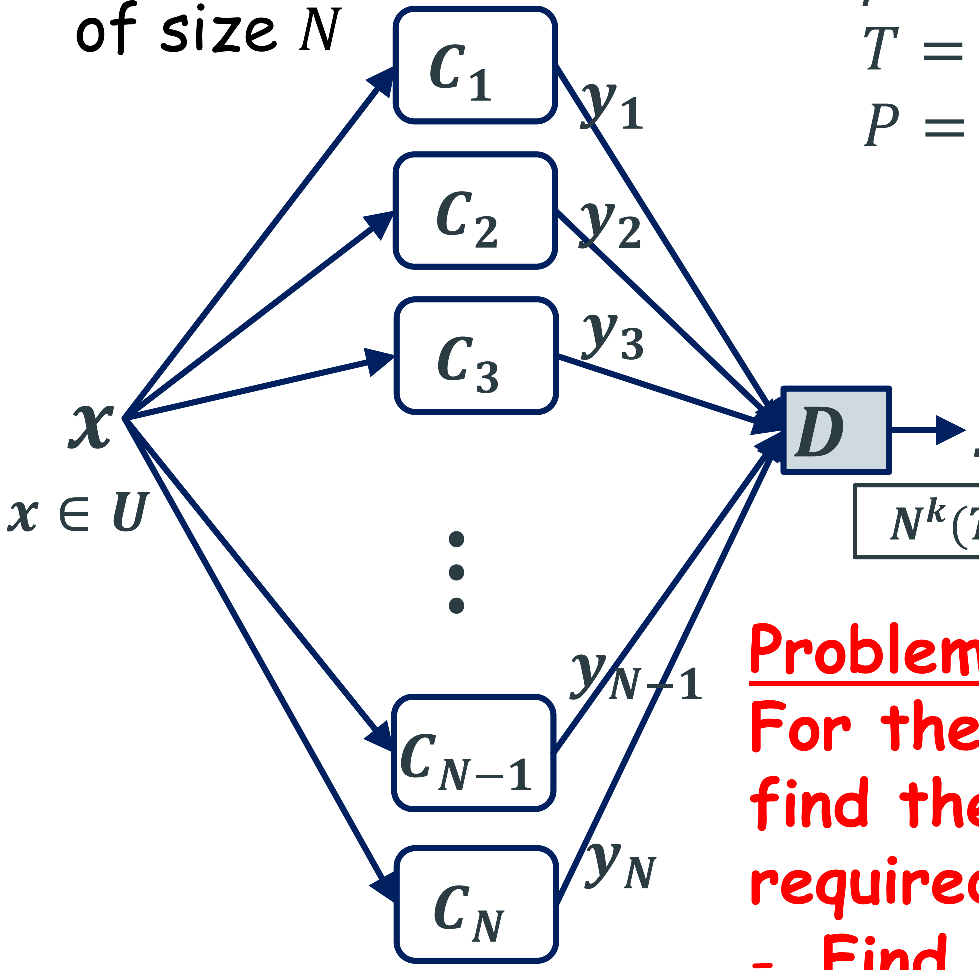
D=Dolecek
G=Gabrys
K=Konstantinova,
La= Langberg,
Le=Levenshtein
M=Molodtsov
Sa=Sala
Si=Siemons,
So=Schoeny
Sw=Schwartz
Y=Yaakobi

Outline

- Problem setup
- Two types of channels
  - General case
  - Substitutions errors
    - Explicit solution
    - Examples
  - Special systems
- ℓ types of channels
- Open problems

Sequence Reconstruction - Problem

A channel system of size N



V-a finite set. $U \subseteq V$.

$\rho: V \times V \rightarrow \mathbb{N}$: a distance function

$T = (t_1, t_2, \dots, t_\ell)$, $t_1 < t_2 < \cdots < t_\ell \in \mathbb{N}$

$P = (p_1, p_2, \dots, p_\ell)$, $0 < p_1 < p_2 < \cdots < p_\ell = 1$

Models:

A (T,P)-channel system: $\boxed{N^u(T,P,U)}$

$\lceil p_i N \rceil$ channels are t_i-error.

A (T,P)-sequenced channel system:

$\boxed{N^k(T,P,U)}$ The first $\lceil p_i N \rceil$ channels are t_i-error

A t-channel system: $\boxed{N(t,U)}$

The average number of errors is t

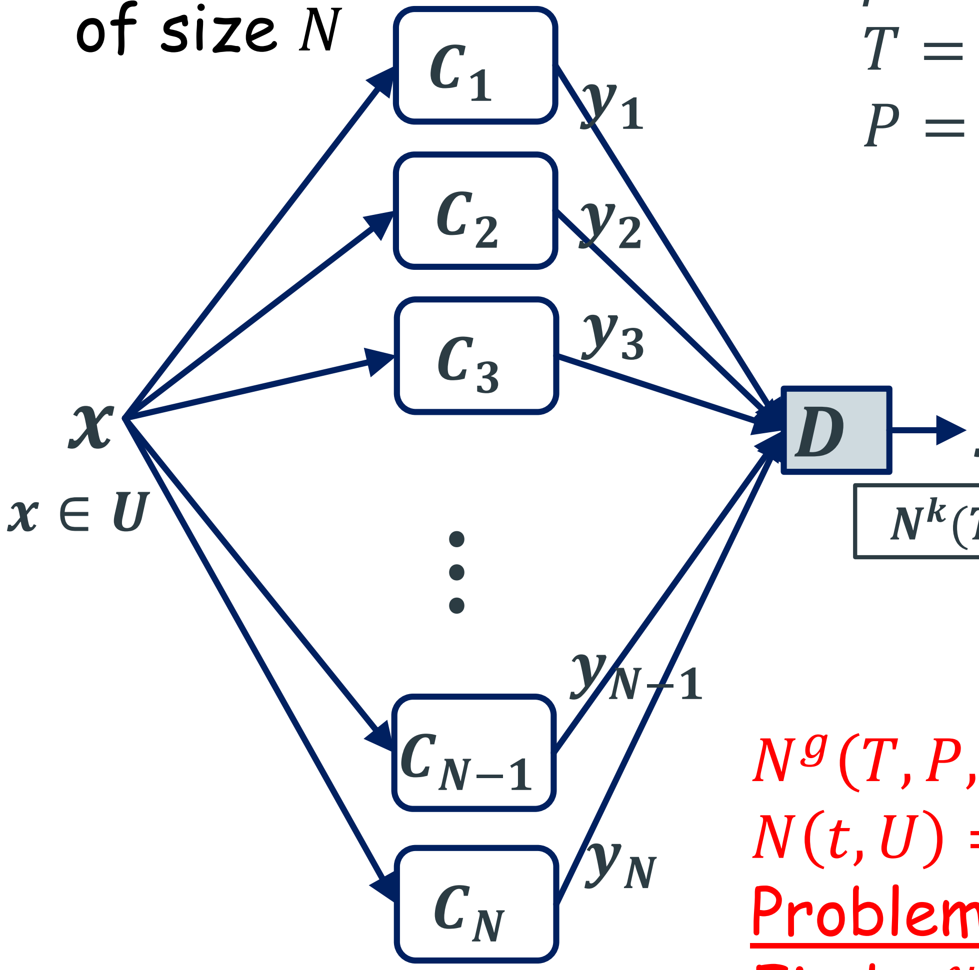
Problem:

For these three models –
find the minimum size of a channel system required for exact reconstruction

- **Find $N^u(T,P,U)$, $N^k(T,P,U)$, $N(t,U)$**
- **Clearly: $N^u(T,P,U) \geq N^k(T,P,U)$**

Sequence Reconstruction - Problem

A channel system of size N



V-a finite set. $U \subseteq V$.

$\rho: V \times V \rightarrow \mathbb{N}$: a distance function

$T = (t_1, t_2, \dots, t_\ell),\ t_1 < t_2 < \cdots < t_\ell \in \mathbb{N}$

$P = (p_1, p_2, \dots, p_\ell),\ 0 < p_1 < p_2 < \cdots < p_\ell = 1$

Models:

A (T,P)-channel system: $N^u(T,P,U)$

$\lceil p_i N \rceil$ channels are t_i-error.

A (T,P)-sequenced channel system:

$N^k(T,P,U)$ The first $\lceil p_i N \rceil$ channels are t_i-error

A t-channel system: $N(t,U)$

The average number of errors is t

$N^g(T,P,U) = max\{N^g(T,P,\{x,z\})\ : x,z \in U\},\ g \in \{u,k\}$

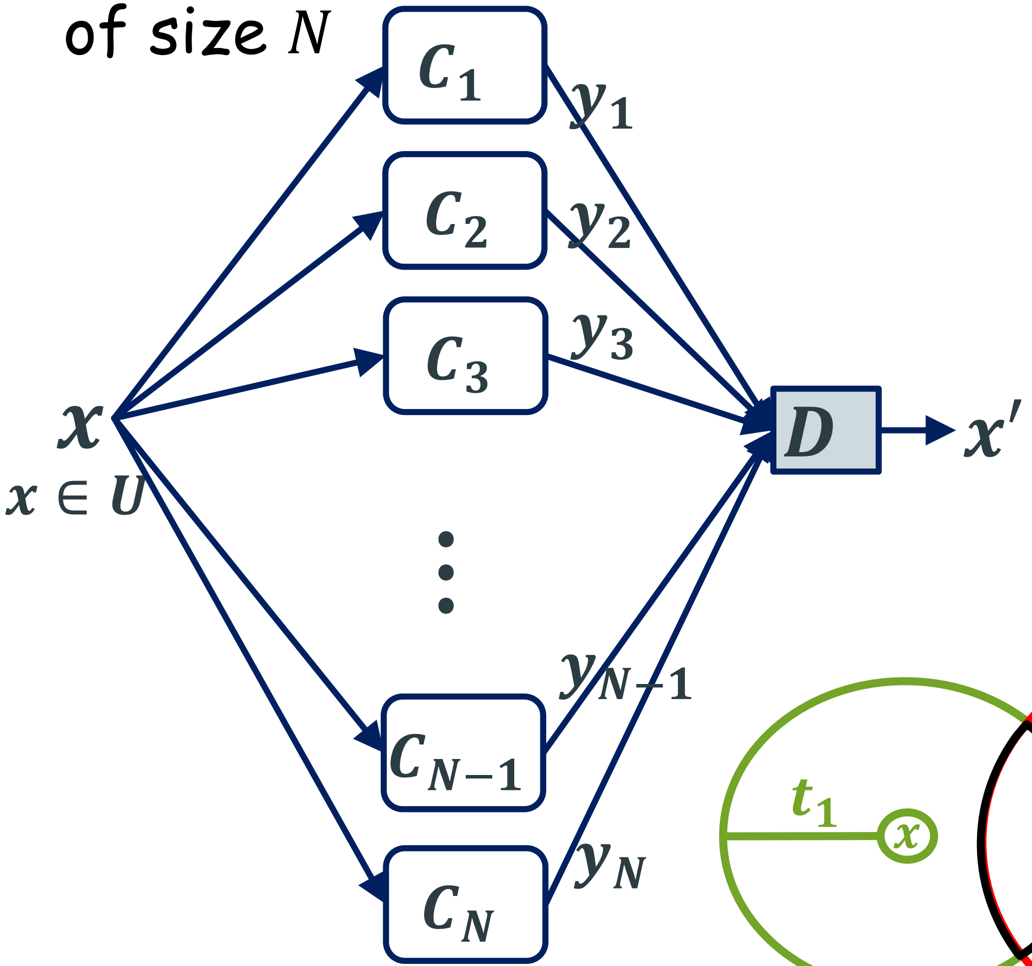
$N(t,U) = max\{N(t,\{x,z\})\ : x,z \in U\}$

<u>Problem</u>: For $U = \{x,z\}$

Find $N^u(T,P,U), N^k(T,P,U), N(t,U)$

Sequence Reconstruction - Notations

A channel system of size N



V : a finite set

$\rho: V \times V \rightarrow \mathbb{N}$: a distance function

$U \subseteq V$

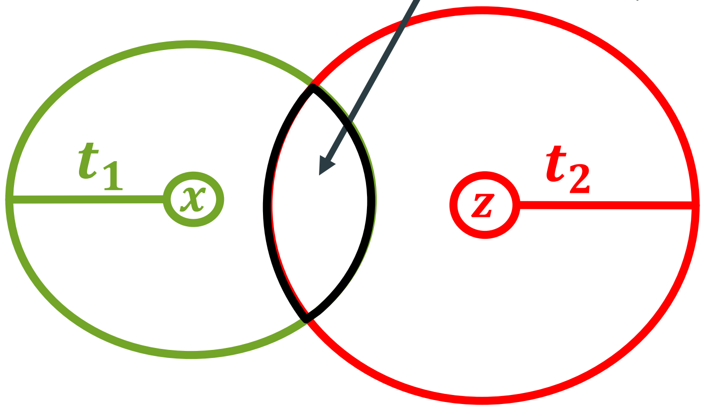
$B_t(x) = \{y : \rho(x, y) \leq t\}$

$I(x, z, t_1, t_2) = B_{t_1}(x) \cap B_{t_2}(z)$

$I(x, z, t) = B_t(x) \cap B_t(z)$

$N(x, z, t_1, t_2) = |I(x, z, t_1, t_2)|$

$N(x, z, t) = |I(x, z, t)|$

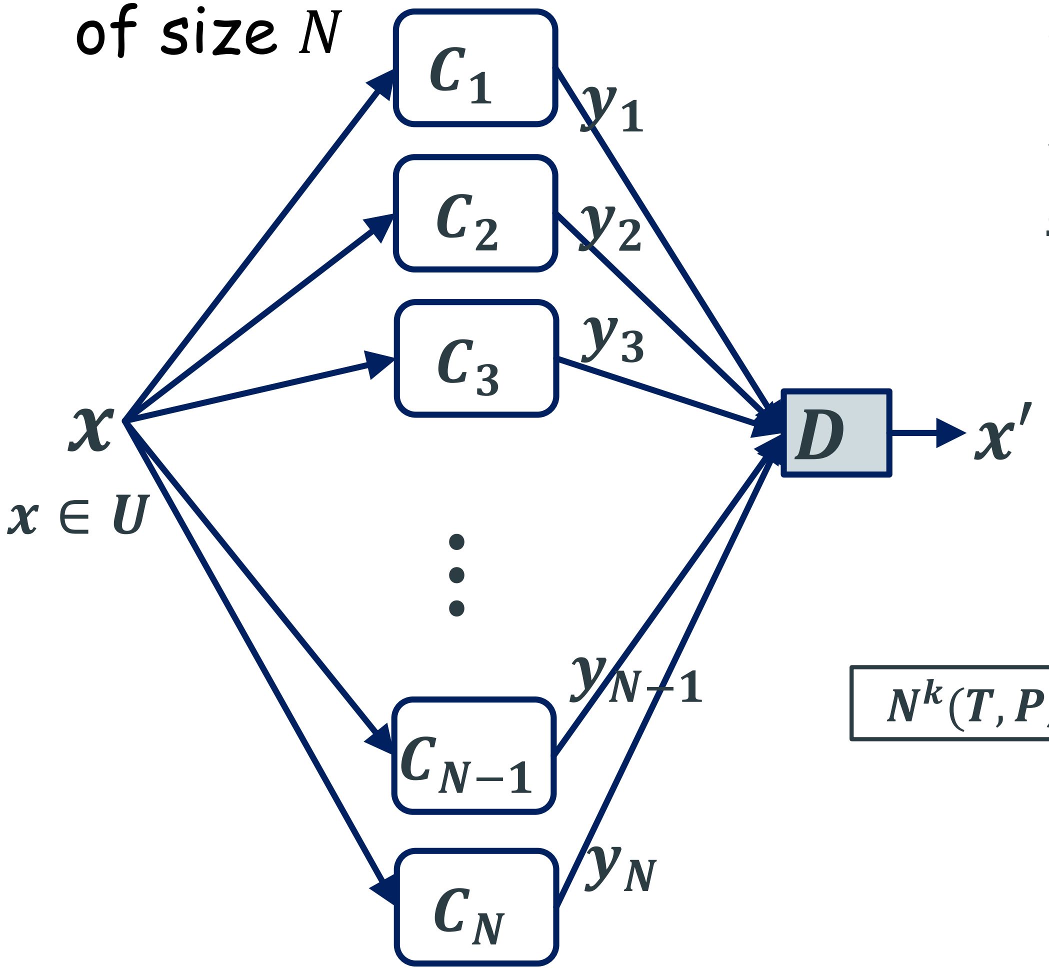


Outline

- Problem setup
- Two types of channels
  - General case
  - Substitutions errors
    - Explicit solution
    - Examples
  - Special systems
- ℓ types of channels
- Open problems

Sequence Reconstruction – $\ell = 2$

A channel system of size N



$U = \{x, z\} \subseteq V, \rho,$
$T = (t_1, t_2),\ t_1 < t_2 \in \mathbb{N}$
$P = (p, 1),\ 0 < p < 1$

Models:

A (T, P)-channel system: $\boldsymbol{N^u(T,P,U)}$

$\lceil pN \rceil$ channels are t_1-error
All channels are t_2-error

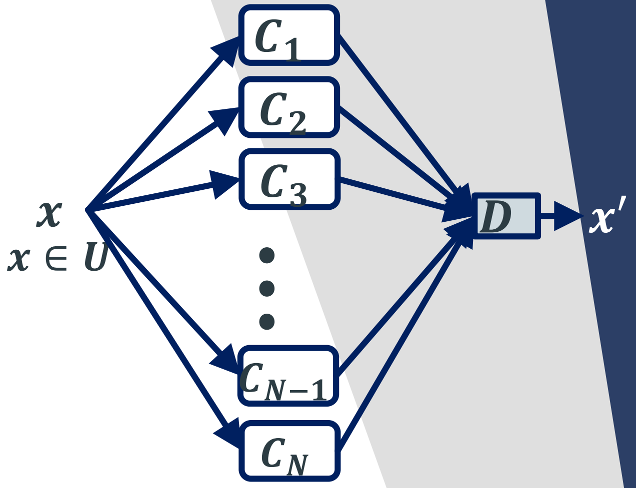
A (T, P)-sequenced channel system:

$\boldsymbol{N^k(T,P,U)}$

The first $\lceil pN \rceil$ channels are t_1-error
All channels are t_2-error

Sequence Reconstruction – $\ell = 2$

Sequenced Model



$x, z \in V, \rho, T = (t_1, t_2), t_1 < t_2 \in \mathbb{N}, P = (p, 1), 0 < p < 1$

A (T, P)-sequenced channel system:

The first $\lceil pN \rceil$ channels are t_1-error

All channels are t_2-error

Problem: $N^k(T, P, \{x, z\})$=?

Solution: $N^k(T, P, \{x, z\}) = L + 1$

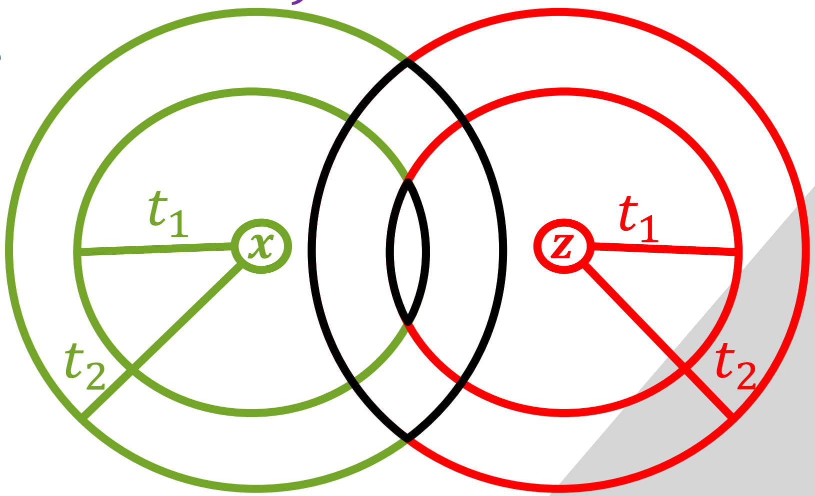
$$L = \min\left\{\left\lfloor \frac{N(x, z, t_1)}{p} \right\rfloor, N(x, z, t_2)\right\}$$

Proof: first part: $N \geq L + 1$ is sufficient:

- $N > N(x, z, t_2)$

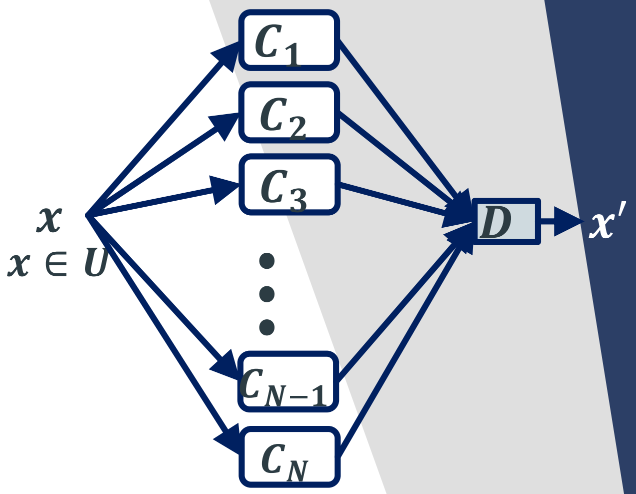
OR

- $N > \left\lfloor \frac{N(x,z,t_1)}{p} \right\rfloor \Longrightarrow \lceil pN \rceil > N(x, z, t_1)$



Sequence Reconstruction – $\ell = 2$

Sequenced Model



$x, z \in V, \rho, T = (t_1, t_2), t_1 < t_2 \in \mathbb{N}, P = (p, 1), 0 < p < 1$

A $(\boldsymbol{T}, \boldsymbol{P})$-sequenced channel system:

The first $\lceil pN \rceil$ channels are t_1-error

All channels are t_2-error

Problem: $N^k(T, P, \{x, z\})$=?

Solution: $N^k(T, P, \{x, z\}) = L + 1$

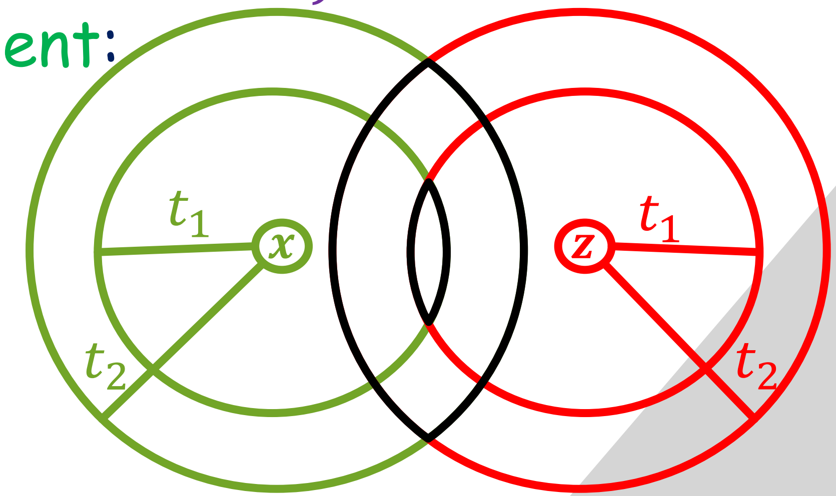
$$L = \min\left\{\left\lfloor\frac{N(x, z, t_1)}{p}\right\rfloor, N(x, z, t_2)\right\}$$

Proof: second part: $N \le L$ is not-sufficient:

- $N \le N(x, z, t_2)$

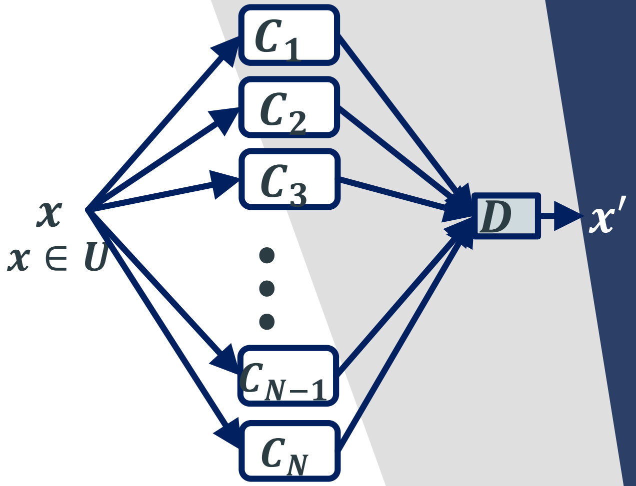
AND

- $N \le \left\lfloor\frac{N(x,z,t_1)}{p}\right\rfloor \Longrightarrow \lceil pN \rceil \le N(x, z, t_1)$



Sequence Reconstruction - $\ell = 2$

Non-Sequenced Model



$x, z \in V, \rho, T = (t_1, t_2), t_1 < t_2 \in \mathbb{N}, P = (p, 1), 0 < p < 1$

A (T, P)-channel system:

$\lceil pN \rceil$ channels are t_1-error

All channels are t_2-error

Problem: $N^u(T, P, \{x, z\})$=?

Solution: $N^u(T, P, \{x, z\}) = L + 1$

$$L = \min\left\{\left\lfloor\frac{N(x,z,t_1,t_2)}{p}\right\rfloor, \left\lfloor\frac{N(z,x,t_1,t_2)}{p}\right\rfloor, N(x, z, t_2), N'(x, z, t_1, p)\right\}$$

$$N'(x, z, t_1, p) = \min\{J : 2\lceil pJ \rceil - J > N(x, z, t_1), J \geq 1\} - 1$$

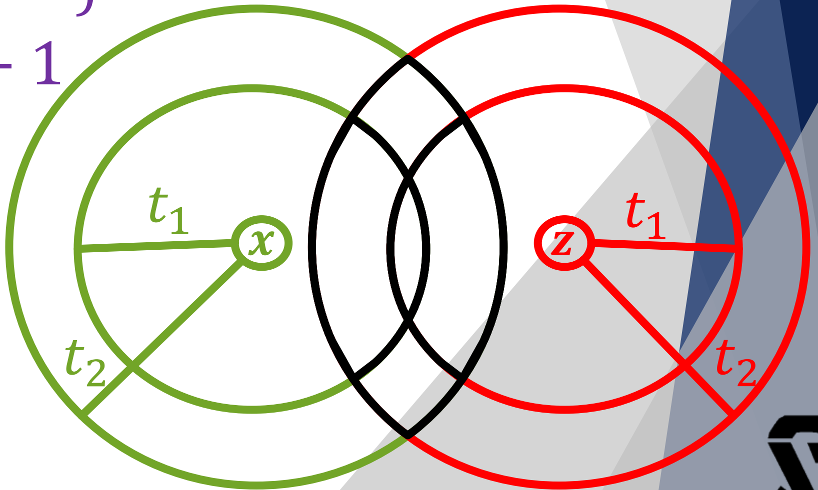
Proof: first part: $N \geq L + 1$ is sufficient:

1) $N > N(x, z, t_2)$ OR

2) $N > \left\lfloor\frac{N(x,z,t_1,t_2)}{p}\right\rfloor \Rightarrow \lceil pN \rceil > N(x, z, t_1, t_2)$ OR

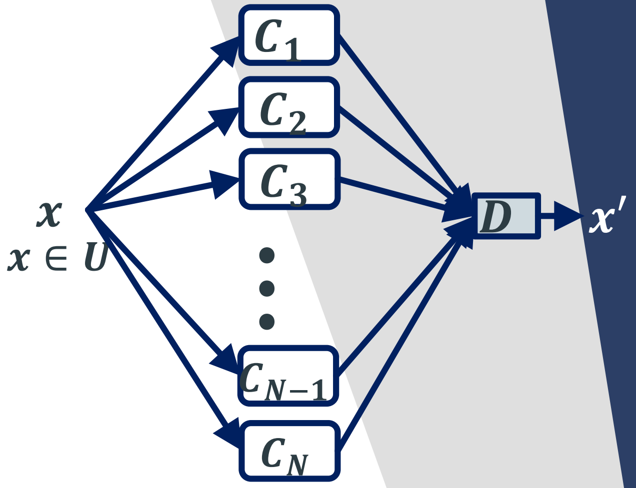
3) $N > \left\lfloor\frac{N(z,x,t_1,t_2)}{p}\right\rfloor \Rightarrow \lceil pN \rceil > N(z, x, t_1, t_2)$ OR

4) $N = L + 1 \Rightarrow 2\lceil pN \rceil - N > N(x, z, t_1) \Rightarrow 2\lceil pN \rceil - N(x, z, t_1) > N$



Sequence Reconstruction - $\ell = 2$

Non-Sequenced Model



$x, z \in V, \rho, T = (t_1, t_2),\ t_1 < t_2 \in \mathbb{N}, P = (p, 1),\ 0 < p < 1$

A (T, P)-channel system:

$\lceil pN \rceil$ channels are t_1-error

All channels are t_2-error

<u>Problem</u>: $N^u(T, P, \{x, z\})$=?

<u>Solution</u>: $N^u(T, P, \{x, z\}) = L + 1$

$$L = \min\left\{\left\lfloor\frac{N(x,z,t_1,t_2)}{p}\right\rfloor, \left\lfloor\frac{N(z,x,t_1,t_2)}{p}\right\rfloor, N(x, z, t_2), N'(x, z, t_1, p)\right\}$$

$$N'(x, z, t_1, p) = \min\{J : 2\lceil pJ \rceil - J > N(x, z, t_1), J \geq 1\} - 1$$

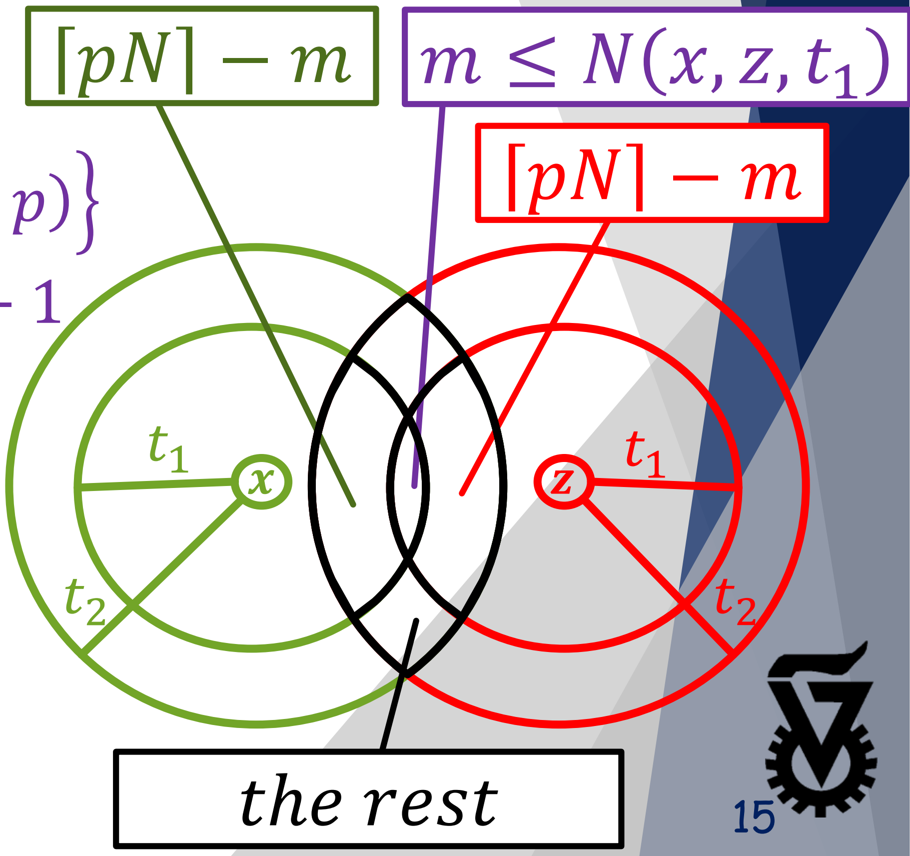
Proof: second part: $N \leq L$ is not sufficient:

1) $N \leq N(x, z, t_2)$ AND

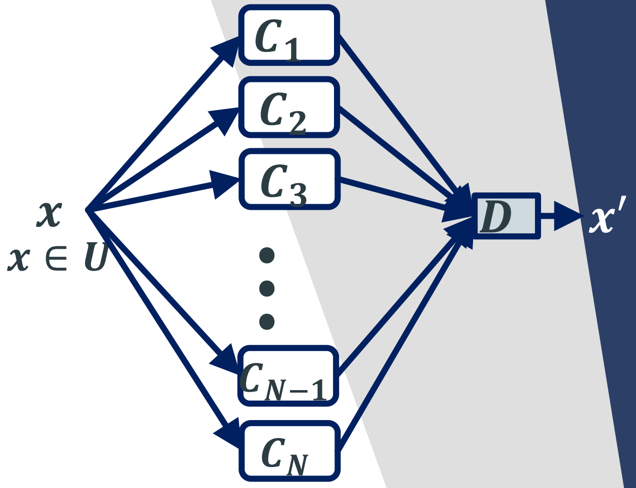
2) $N \leq \left\lfloor\frac{N(x,z,t_1,t_2)}{p}\right\rfloor \Rightarrow \lceil pN \rceil \leq N(x, z, t_1, t_2)$ AND

3) $N \leq \left\lfloor\frac{N(z,x,t_1,t_2)}{p}\right\rfloor \Rightarrow \lceil pN \rceil \leq N(z, x, t_1, t_2)$ AND

4) $N \leq N'(x, z, t_1, p) \Rightarrow 2\lceil pN \rceil - N(x, z, t_1) \leq N$



Sequence Reconstruction – $\ell = 2$
Summarize – a (T,P)-Channel System



$x, z \in V, \rho, T = (t_1, t_2),\ t_1 < t_2 \in \mathbb{N}, P = (p, 1),\ 0 < p < 1$

A (T,P)-channel system:

$\lceil pN \rceil$ channels are t_1-error

All channels are t_2-error

$N^u(T, P, \{x, z\}) = L + 1$

$$L = \min\left\{\left\lceil \frac{N(x,z,t_1,t_2)}{p} \right\rceil, \left\lceil \frac{N(z,x,t_1,t_2)}{p} \right\rceil, N(x,z,t_2), N'(x,z,t_1,p)\right\}$$

$$N'(x,z,t_1,p) = \min\{J : 2\lceil pJ \rceil - J > N(x,z,t_1), J \geq 1\} - 1$$

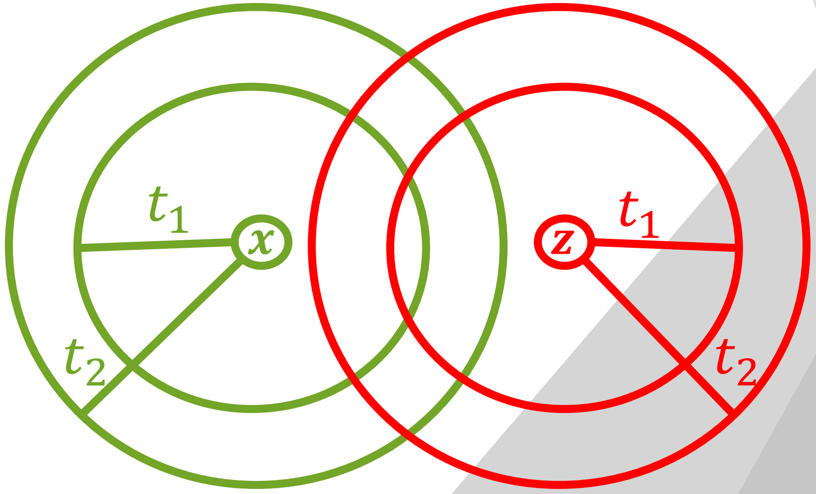
A (T,P)-sequenced channel system:

The first $\lceil pN \rceil$ channels are t_1-error

All channels are t_2-error

$N^k(T, P, \{x, z\}) = L + 1$

$$L = \min\left\{\left\lceil \frac{N(x,z,t_1)}{p} \right\rceil, N(x,z,t_2)\right\}$$



Outline

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- Open problems

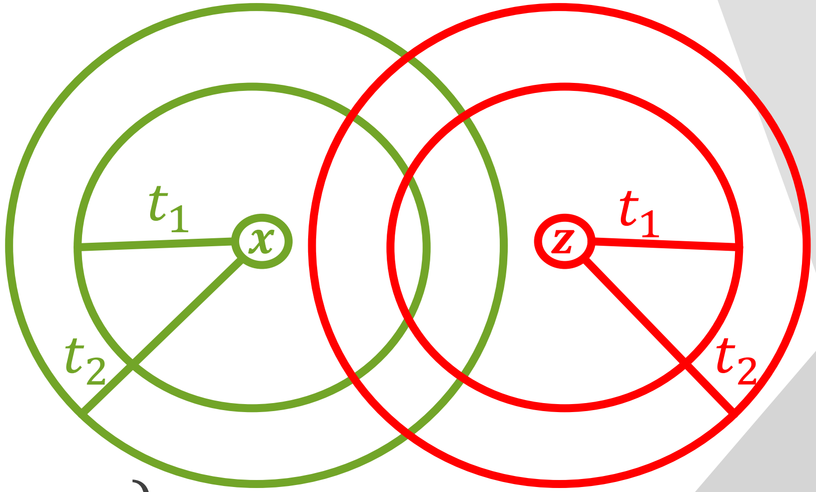
Sequence Reconstruction – $\ell = 2$ Substitution (Hamming)

$x, z \in V,\ \rho, d = \rho(x,z),\ T = (t_1, t_2), t_1 < t_2 \in \mathbb{N}, P = (p, 1), 0 < p < 1$

$\lceil pN \rceil$ channels are t_1-error, all channels are t_2-error.

- $N(d, t_1, t_2) = N(x, z, t_1, t_2)$
- $N^g(T, P, d) = N^g(T, P, \{x, z\})$
- $N^g(T, P, d) \geq N^g(T, P, d+1)$
- $N^k(T, P, d) = \min\left\{\left\lceil \frac{N(d,t_1)}{p} \right\rceil, N(d, t_2)\right\}$
- $N^u(T, P, d) = \min\left\{\left\lceil \frac{N(d,t_1,t_2)}{p} \right\rceil, N(d, t_2), N'(d, t_1, p)\right\}$

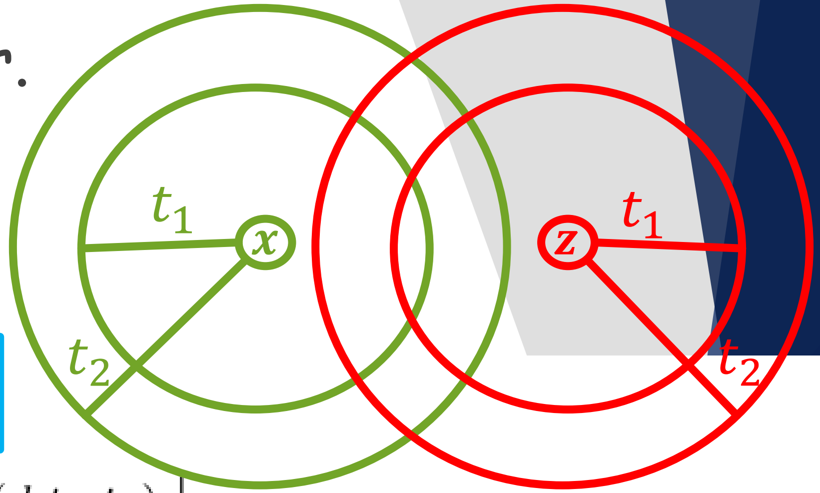
$$N'(d, t_1, p) = \min\{J : 2\lceil pJ \rceil - J > N(d, t_1), J \geq 1\}$$



Sequence Reconstruction – $\ell = 2$
Substitution (Hamming) – Non-Sequenced Model

$x, z \in V,\ \rho, d = \rho(x,z),\ T = (t_1, t_2),\ t_1 < t_2 \in \mathbb{N}, P = (p, 1),\ 0 < p < 1$

$\lceil pN \rceil$ channels are t_1-error, all channels are t_2-error.



$$N^u(T,P,d) = \min\left\{\left\lceil \frac{N(d,t_1,t_2)}{p} \right\rceil, N(d,t_2), N'(d,t_1,p)\right\}$$

$$N'(d,t_1,p) = \min\{J : 2\lceil pJ \rceil - J > N(d,t_1), J \geq 1\}$$

- $\boxed{\textit{For } 0 < p \leq 1/2:}$

$$N^u(T,P,d) = \begin{cases} 1 & \textit{if } d > 2t_1 \\ N(d,t_2)+1 & \textit{otherwise,} \\ & \textit{and } t_2 = \\ \boxed{\lfloor N(d,t_1,t_2)/p \rfloor + 1} & \textit{otherwise.} \end{cases}$$

$$\boxed{N^u(T,P,d) = \begin{cases} 1 & \textit{if } d > 2t_1, \\ \Theta(n^{\lfloor \frac{t_1+t_2-d}{2} \rfloor}) & \textit{otherwise.} \end{cases}}$$

- $\boxed{\textit{For } 1/2 < p < 1:}$

$$N^u(T,P,d) = \begin{cases} \left\lceil \frac{N(d,t_1,t_2)}{p} \right\rceil & \textit{if } d \textit{ is even}, t_2 = t_1+1, \\ & \textit{and } \left(\left(\frac{1}{2} < p \leq \frac{2}{3}\right) \vee \right. \\ & \left.\left(\frac{2}{3} < p < \frac{3}{4} \wedge d < \frac{2-2p}{3p-2}\right)\right), \\ \boxed{N'(d,t_1,p)} & \textit{otherwise.} \end{cases}$$

$$\boxed{N^u(T,P,d) = \Theta(n^{\lfloor \frac{2t_1-d}{2} \rfloor})}$$

Sequence Reconstruction – $\ell = 2$
Substitution (Hamming) – Non-Sequenced Model

$x, z \in V,\ \rho, d = \rho(x,z),\ T = (t_1, t_2),\ t_1 < t_2 \in \mathbb{N}, P = (p, 1),\ 0 < p < 1$

$\lceil pN \rceil$ channels are t_1-error, all channels are t_2-error.

$N^u(T,P,d) = \min\left\{\left\lceil \frac{N(d,t_1,t_2)}{p} \right\rceil, N(d,t_2), N'(d,t_1,p)\right\}$

$N'(d,t_1,p) = \min\{J : 2\lceil pJ \rceil - J > N(d,t_1), J \geq 1\}$

For $1/2 < p < 1$:

$$N^u(T,P,d) = \Theta(n^{\lfloor \frac{2t_1-d}{2} \rfloor})$$

For $0 < p \leq 1/2$:

$$N^u(T,P,d) = \begin{cases} 1 & \text{if } d > 2t_1, \\ \Theta(n^{\lfloor \frac{t_1+t_2-d}{2} \rfloor}) & \text{otherwise.} \end{cases}$$

	All channels are identical	A (T,P)-channel system
$0 < p \leq \frac{1}{2}, d = 1, T = (2,4)$	$\Theta(n^3)$	$\Theta(n^2)$
$0 < p \leq \frac{1}{2}, d = 1, T = (2,8)$	$\Theta(n^7)$	$\Theta(n^4)$
$\frac{1}{2} < p \leq \frac{2}{3}, d = 2, T = (4,5)$	$\Theta(n^4)$	$\Theta(n^3)$

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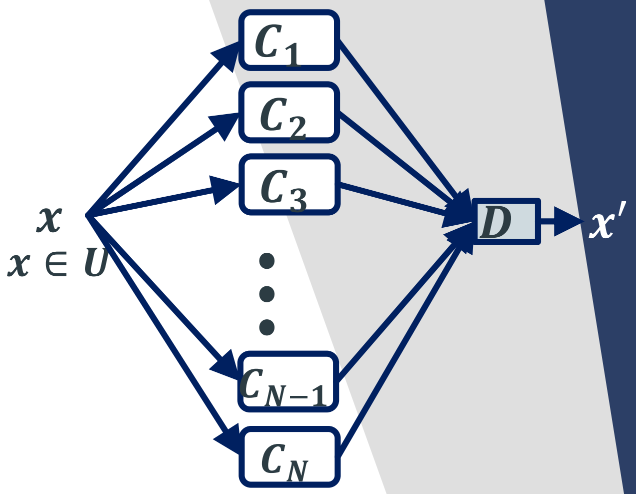
Sequence Reconstruction – A (T, i, b)-Channel System



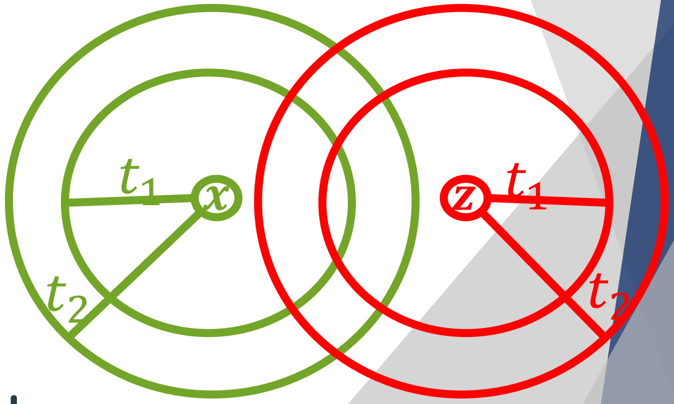
$x, z \in V, \rho, T = (t_1, t_2), t_1 < t_2 \in \mathbb{N}, b \in \mathbb{N}, i \in \{1,2\}$

A (T, i, b)-sequenced channel system:

The first b channels are t_i-error

All channels are t_{3-i}-error

$N^k(T, i, b, \{x, z\})$ – minimum size of a (T, i, b)-sequenced channel system required for exact reconstruction



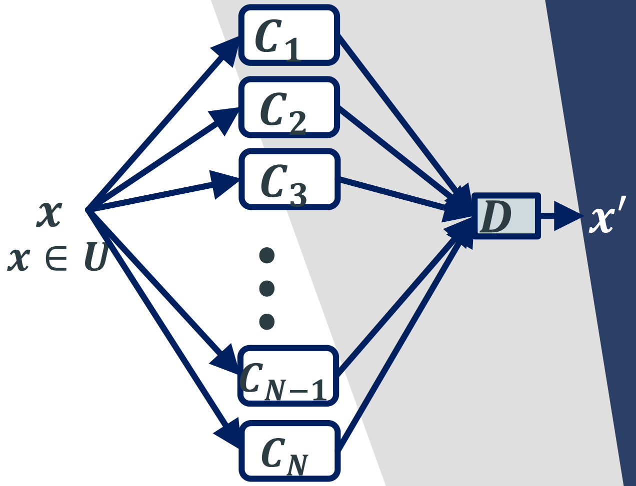
A (T, i, b)-channel system:

b channels are t_i-error

All channels are t_{3-i}-error

$N^u(T, i, b, \{x, z\})$ – minimum size of a (T, i, b)-channel system required for exact reconstruction

Sequence Reconstruction
A $(T, 1, b)$-Channel System



$x, z \in V, \rho, T = (t_1, t_2), t_1 < t_2 \in \mathbb{N}, b \in \mathbb{N}$

A $(T, 1, b)$-sequenced channel system:

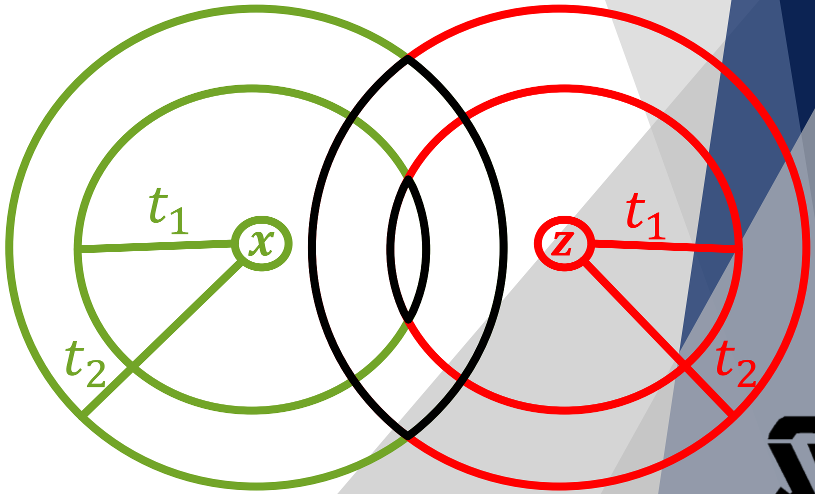
The first b channels are t_1-error

All channels are t_2-error

A $(T, 1, b)$-channel system:

b channels are t_1-error

All channels are t_2-error

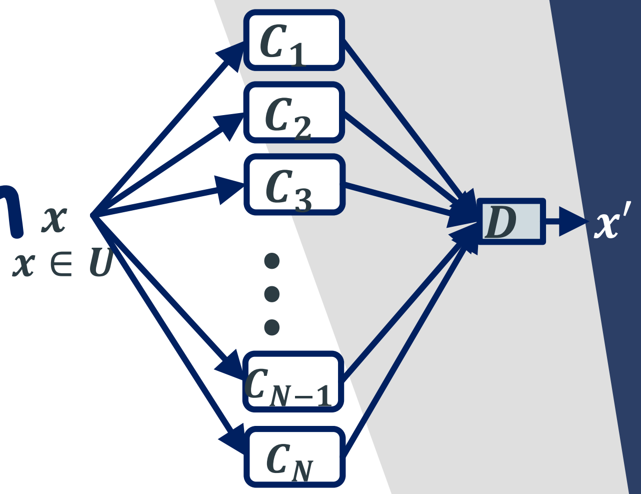


Problem: $N^k(T, 1, b, \{x, z\})$=?, $N^u(T, 1, b, \{x, z\})$=?

Solution: $N^k(T, 1, b, \{x, z\}) = N^u(kT, 1, b, \{x, z\}) = L + 1$

$$L = \begin{cases} N(x, z, t_2), & if\ N(x, z, t_1) \geq b \\ N(x, z, t_1), & else \end{cases}$$

Sequence Reconstruction
A $(T,2,b)$-Sequenced Channel System



$x, z \in V, \rho, T = (t_1, t_2), t_1 < t_2 \in \mathbb{N}, b \in \mathbb{N}$

A $(T,2,b)$-sequenced channel system:

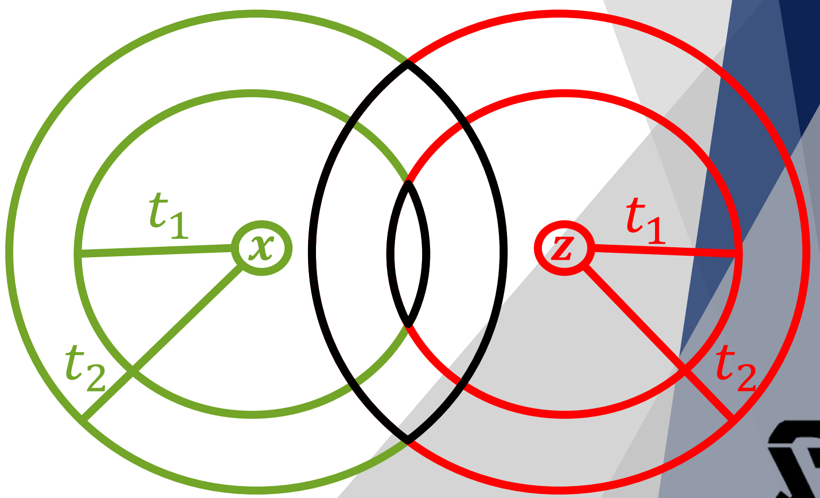
The first b channels are t_2-error

All channels are t_1-error

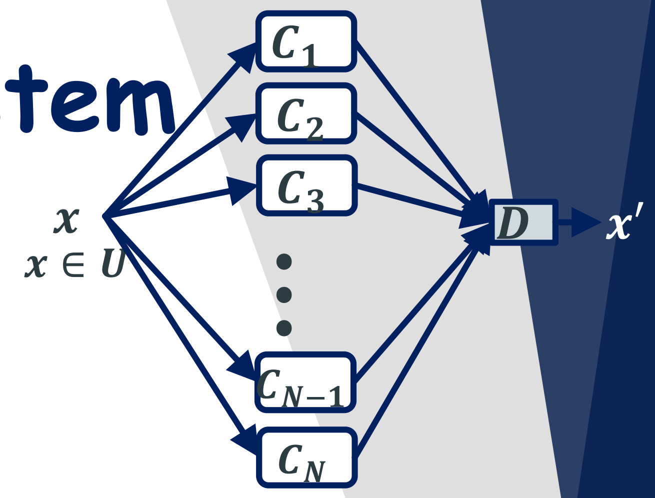
Problem: $N^k(T,2,b,\{x,z\})$=?

Solution: $N^k(T,2,b,\{x,z\}) = L + 1$

$L = \min\{ N(x,z,t_1) + b, N(x,z,t_2)\}$



Sequence Reconstruction
A $(T,2,b)$-Non-Sequenced Channel System



$x, z \in V, \rho, T = (t_1, t_2), t_1 < t_2 \in \mathbb{N}, b \in \mathbb{N}$

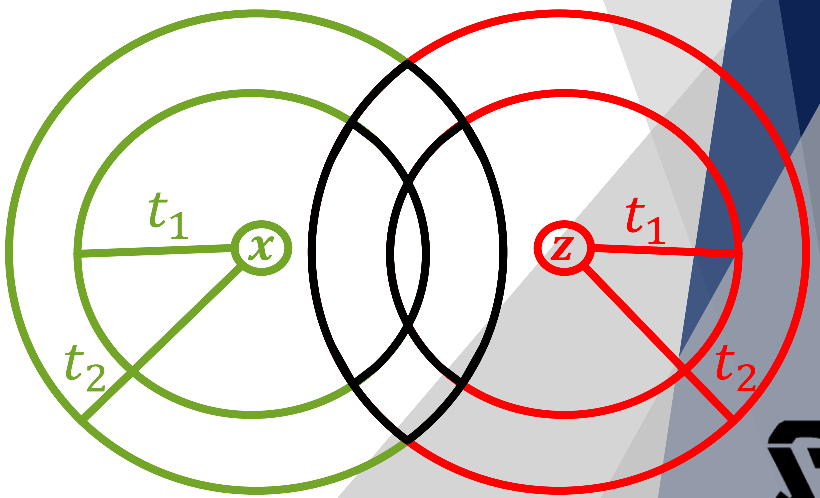
A $(T,2,b)$-channel system:

- b channels are t_2-error
- All channels are t_1-error

Problem: $N^u(T,2,b,\{x,z\})$=?

Solution: $N^u(T,2,b,\{x,z\}) = L + 1$

$$L = \min \begin{Bmatrix} N(x,z,t_1,t_2) + b, \\ N(z,x,t_1,t_2) + b, \\ N(x,z,t_2), \\ N(x,z,t_1) + 2b \end{Bmatrix}$$



Sequence Reconstruction – $\ell = 2$

$x, z \in V, \rho, T = (t_1, t_2), t_1 < t_2 \in \mathbb{N}, P = (p, 1), 0 < p < 1, b \in \mathbb{N}.$

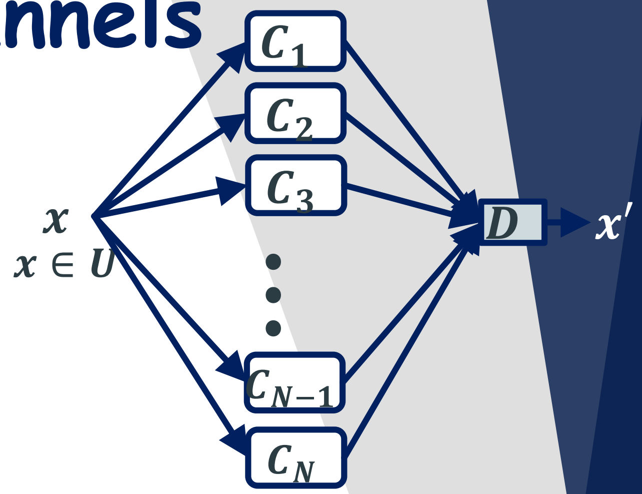
Solution: $N = L + 1$, **where** $L =$

	Sequenced	Non-Sequenced
A (T, P) -channel system	$\min\left\{\left\lceil\frac{N(x,z,t_1)}{p}\right\rceil, N(x,z,t_2)\right\}$	$\min\left\{\begin{array}{c}\left\lceil\frac{N(x,z,t_1,t_2)}{p}\right\rceil \\ \left\lceil\frac{N(z,x,t_1,t_2)}{p}\right\rceil, \\ N(x,z,t_2), N'(x,z,t_1,p)\end{array}\right\}$
A $(T, 1, b)$ -channel system	$\begin{cases} N(x,z,t_2), & \text{if } N(x,z,t_1) \geq b \\ N(x,z,t_1), & \text{else} \end{cases}$	$\begin{cases} N(x,z,t_2), & \text{if } N(x,z,t_1) \geq b \\ N(x,z,t_1), & \text{else} \end{cases}$
A $(T, 2, b)$ -channel system	$\min\{N(x,z,t_1) + b, N(x,z,t_2)\}$	$\min\left\{\begin{array}{c} N(x,z,t_1,t_2) + b, \\ N(z,x,t_1,t_2) + b, \\ N(x,z,t_2), \\ N(x,z,t_1) + 2b \end{array}\right\}$

Outline

- Problem setup
- Two types of channels
  - General case
  - Substitutions errors
    - Explicit solution
    - Examples
  - Special systems
- ℓ types of channels
- Open problems

Sequence Reconstruction – ℓ Types of Channels Sequenced Model

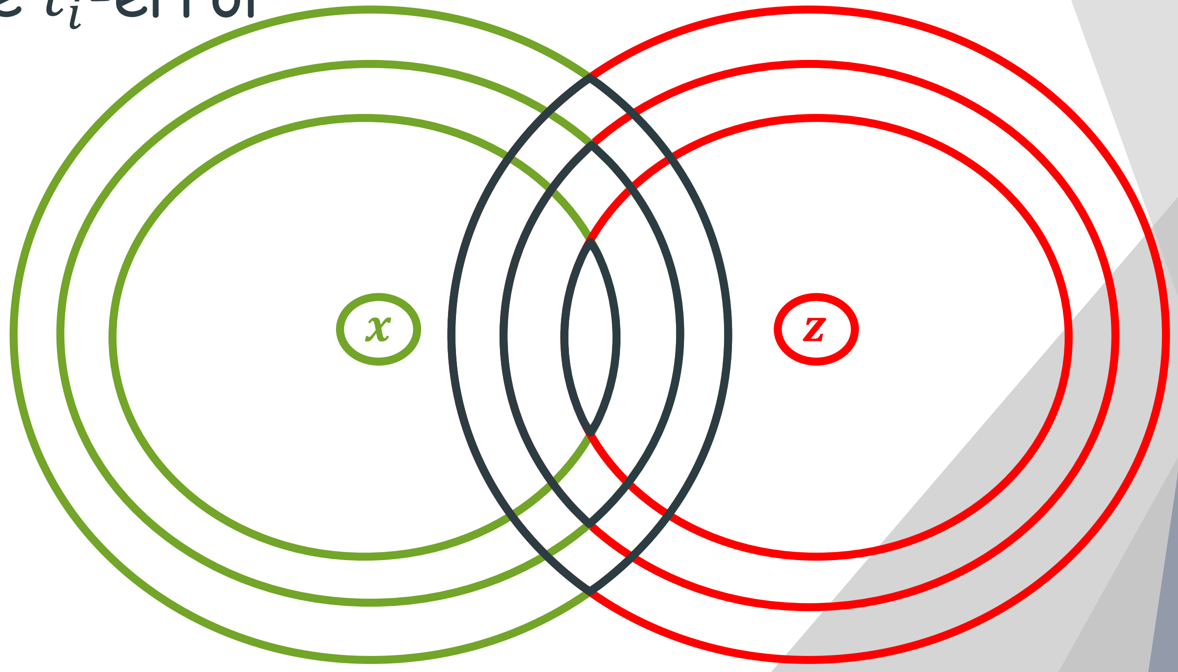


$x, z \in V, \rho,$
$T = (t_1, t_2, \dots, t_\ell),\ t_1 < t_2 < \cdots < t_\ell \in \mathbb{N},$
$P = (p_1, p_2, \dots, p_\ell),\ 0 < p_1 < p_2 < \cdots < p_\ell = 1$

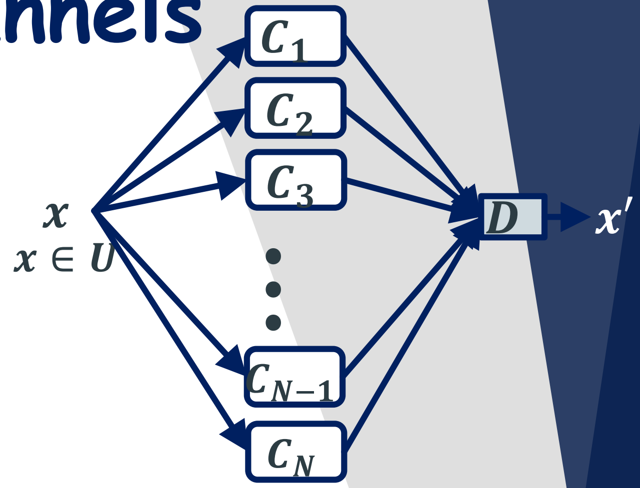
A (T, P)-sequenced channel system:

The first $\lceil p_i N \rceil$ channels are t_i-error

$$N^k(T, P, U) = L + 1$$
$$L = \min\left\{\left\lfloor \frac{N(x, z, t_i)}{p_i} \right\rfloor : 1 \le i \le \ell\right\}$$



Sequence Reconstruction – ℓ Types of Channels Non-Sequenced Model



$x, z \in V, \rho,$

$T = (t_1, t_2, \dots, t_\ell),\ t_1 < t_2 < \cdots < t_\ell \in \mathbb{N},$

$P = (p_1, p_2, \dots, p_\ell),\ 0 < p_1 < p_2 < \cdots < p_\ell = 1$

A (T, P)-channel system:

$\lceil p_i N \rceil$ channels are t_i-error

$N^u(T, P, U) = \min\{L_1, L_2, L_3\} + 1$

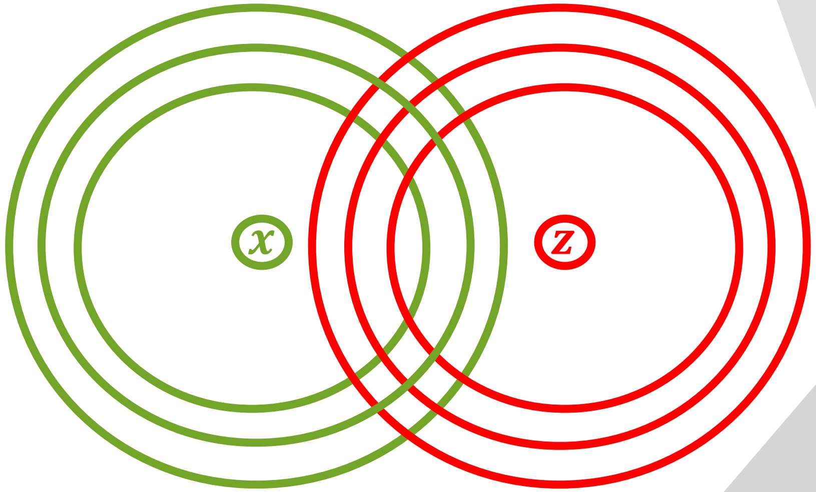
$$L_1 = \min\left\{\left\lceil \frac{N(x, z, t_i, t_\ell)}{p_i} \right\rceil : 1 \le i < \ell\right\}$$

$$L_2 = \min\left\{\left\lceil \frac{N(z, x, t_i, t_\ell)}{p_i} \right\rceil : 1 \le i < \ell\right\}$$

$$L_2 = N(x, z, t_\ell)$$

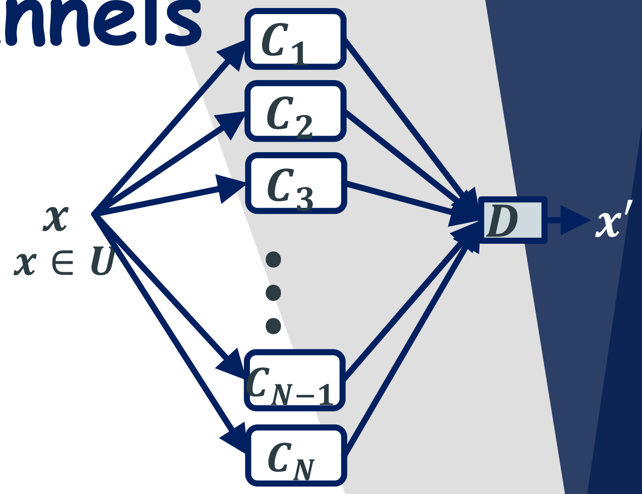
$$L_4 = \min\{N'(x, z, t_i, t_j, p_i, p_j) : 1 \le i, j < \ell\}$$

$$N'(x, z, t_i, t_j, p_i, p_j) = \min\{J : \lceil p_i J \rceil + \lceil p_i J \rceil - J > N(x, z, t_i, t_j), J \ge 1\} - 1$$



Sequence Reconstruction – ℓ Types of Channels

A t-channel system



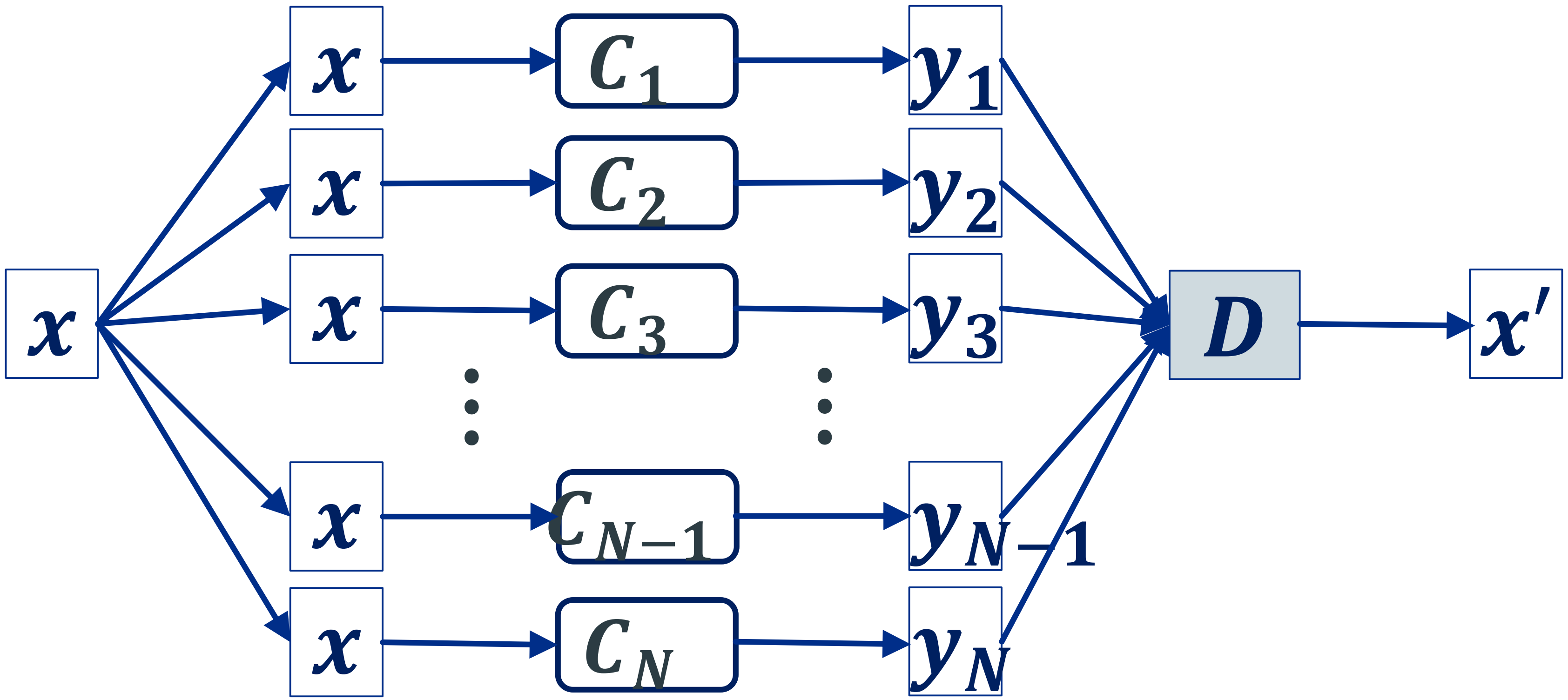
$x, z \in V, \rho, t \in \mathbb{N}, d = \rho(x, z)$

A t-channel system:

The average number of errors is t

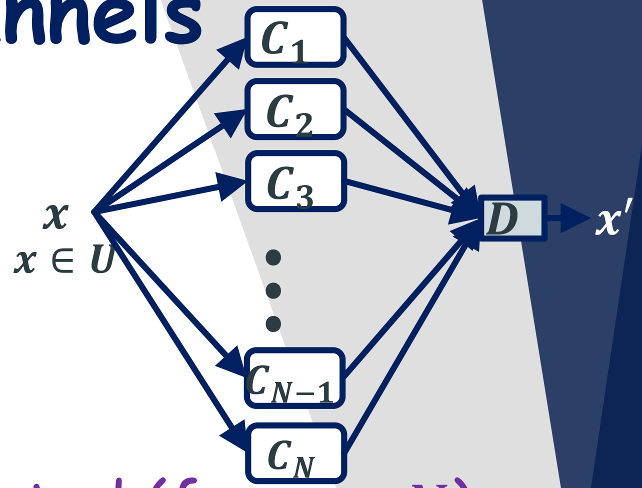
Solution: If $d > 2t$ then $N(t, U) = 1$.

Otherwise, exact reconstruction is not supported (for any N).



Sequence Reconstruction – ℓ Types of Channels

A t-channel system



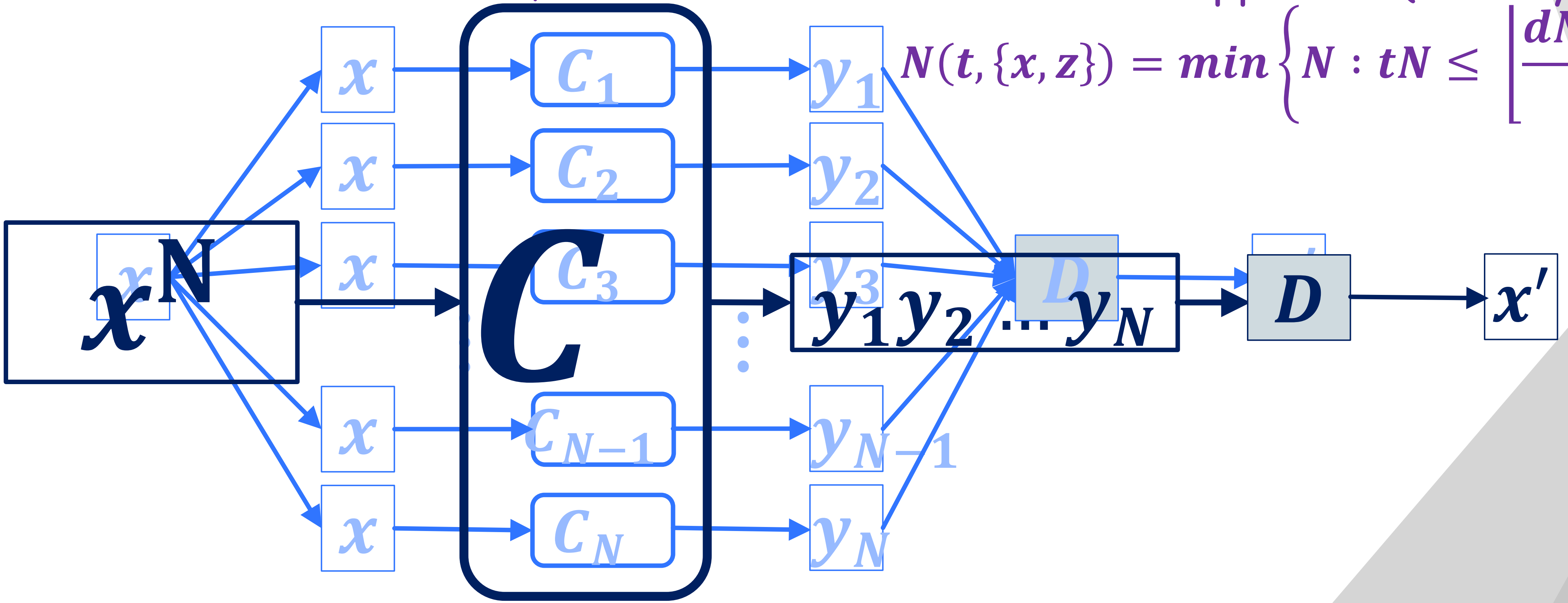
$x, z \in V, \rho, t \in \mathbb{N}, d = \rho(x, z)$

A t-channel system:

The average number of errors is t

Solution: If $d > 2t$ then $N(t, U) = 1$.
Otherwise, exact reconstruction is not supported (for any N).

$$N(t, \{x, z\}) = min\left\{N : tN \leq \left\lfloor \frac{dN-1}{2} \right\rfloor\right\}$$



Outline

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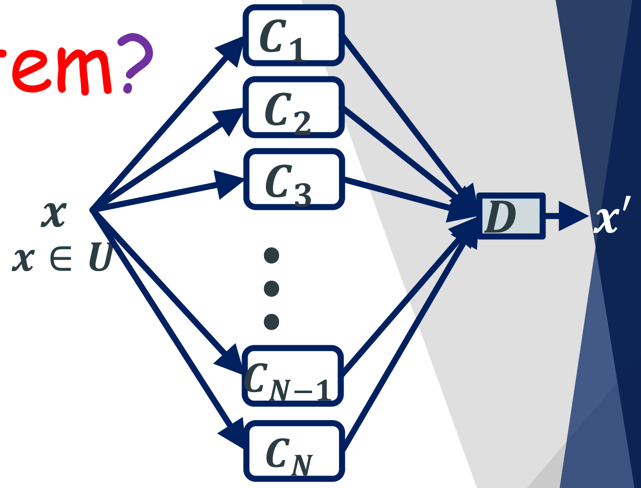
Sequence Reconstruction for non-identical Channels

Open Problems

Given: U, channels types.

Q: What is the minimal size of a channel system?

- Combination of types of errors.
- Other data about the number of errors.



Given: A channel system of size N.

Q: What is the minimal distance of the input?

- Combination of types of errors.
- Some data about the number of errors.

Thank You!